We investigate some properties of \( * \)-concave and convex multifunctions on the real line with convex bounded closed values. In particular we consider the Hadamard inequality and the Hardy–Littlewood–Pólya majorization theory in the case of multifunctions.

1. Basic definitions

Let \( X \) be a real Banach space. Denote by \( clb(X) \) the set of all nonempty bounded closed convex subsets of \( X \). For given \( A,B \in clb(X) \) and \( \lambda \geq 0 \) we define \( A + B = \{ a + b : a \in A, b \in B \} \), \( \lambda A = \{ \lambda a : a \in A \} \),

\[
A^* + B = cl(A + B) = cl(clA + clB).
\]

The structure \( (clb(X), +) \) is an Abelian semigroup with the neutral element \( \{0\} \). It is clear that

\[
\lambda(A + B) = \lambda A + \lambda B, \quad (\lambda + \mu)A = \lambda A + \mu A, \quad \lambda(\mu A) = \lambda\mu A, \quad 1 \cdot A = A
\]

for all \( \lambda, \mu \geq 0 \) and \( A,B \in clb(X) \). Thus the triple \( (clb(X), +, \cdot) \) is also an abstract convex cone (for definition see e.g. [11]). Since

\[
A^* + C = B^* + C \implies A = B
\]

(cf. [11]), the cancellation law is satisfied.

Let \( d \) be the Hausdorff metric in \( clb(X) \) derived from the norm \( \| \cdot \| \) in \( X \), i.e. \( d(A,B) = \max \{ e(A,B), e(B,A) \} \), where \( e(A,B) = \sup_{a \in A} \rho(a,B) \) and \( \rho(a,B) = \inf_{b \in B} ||a - b|| \) for \( A,B \in clb(X) \). For given \( A \in clb(X) \) we define \( ||A|| = \sup \{ ||a|| : a \in A \} = d(A, \{0\}) \). The metric space \( (clb(X), d) \) is complete (see e.g. [1, Theorem II-3, p. 40]). Moreover \( d \) is translation invariant since

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\[ d(A + C, B + C) = d(A + C, B + C) = d(A, B) \]

and positively homogeneous

\[ d(\lambda A, \lambda B) = \lambda d(A, B) \]

for all \( A, B, C \in clb(X) \) (cf. [2, Lemma 2.2]).

A multifunction \( F: [a, b] \longrightarrow clb(X) \) is said to be \(*\)-concave (\(*\)-convex) if

\[
F(\lambda x + (1 - \lambda)y) \subset \lambda F(x) + (1 - \lambda)F(y),
\]

\[
(\lambda F(x) + (1 - \lambda)F(y) \subset F(\lambda x + (1 - \lambda)y) )
\]

for all \( x, y \in [a, b] \) and \( \lambda \in (0, 1) \).

**Remark 1**

The concavity of multifunctions, defined as follows,

\[ F(\lambda x + (1 - \lambda)y) \subset \lambda F(x) + (1 - \lambda)F(y), \quad x, y \in [a, b], \lambda \in (0, 1) \]

implies the \(*\)-concavity, but not conversely. To see this we consider two sets \( A, B \in clb(X) \) such that \( A + B \neq cl(A + B) \) (an example could be found in [10, pp. 712-713]) and the multifunction \( F: [0, 1] \longrightarrow clb(X) \) given by the formula \( F(t) = tA + (1-t)B \). It is easy to check that \( F(\lambda t + (1 - \lambda)s) \subset \lambda F(t) + (1 - \lambda)F(s) \) for all \( t, s \in [0, 1] \) and \( \lambda \in (0, 1) \) but

\[ F\left(\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1\right) = F\left(\frac{1}{2}\right) = \frac{1}{2}(A + B) \not\subset \frac{1}{2}(A + B) = \frac{1}{2}[F(0) + F(1)]. \]

**Remark 2**

A multifunction \( F: [a, b] \longrightarrow clb(X) \) is \(*\)-convex if and only if it is convex i.e.,

\[ \lambda F(x) + (1 - \lambda)F(y) \subset F(\lambda x + (1 - \lambda)y), \quad x, y \in [a, b], \lambda \in (0, 1). \]

We note that every convex multifunction with non-empty values has convex values. Indeed, \( \lambda F(x) + (1 - \lambda)F(x) \subset F(x) \) for all \( \lambda \geq 0 \) and \( x \in [a, b] \).

A multifunction \( F: [a, b] \longrightarrow clb(X) \) is said to be increasing if \( F(x) \subset F(y) \) whenever \( x, y \in [a, b] \) and \( x < y \).

A set \( \Delta = \{y_0, y_1, \ldots, y_n\} \), where \( a = y_0 < y_1 < \ldots < y_n = b \), is said to be a partition of \([a, b]\). For given partition \( \Delta \) we set \( \delta(\Delta) = \max \{y_i - y_{i-1}: \ i = 1, \ldots, n\} \). For the partition \( \Delta \) and for a system \( \tau = (\tau_1, \ldots, \tau_n) \) of intermediate points \( \tau_i \in [y_{i-1}, y_i] \) we create the Riemann sum

\[ S(\Delta, \tau) = (y_1 - y_0)F(\tau_1) + \cdots + (y_n - y_{n-1})F(\tau_n). \]
If for every sequence \((\Delta^\nu)\) of partitions \(\Delta^\nu = \{y_0^\nu, y_1^\nu, \ldots, y_n^\nu\}\) of \([a, b]\) such that \(\lim_{\nu \to \infty} \delta(\Delta^\nu) = 0\), and for every sequence \((\tau^\nu)\) of systems of intermediate points, the sequence of the Riemann sums \(\{S(\Delta^\nu, \tau^\nu)\}\) tends to the same limit \(I \in clb(X)\), then \(F\) is said to be \textit{Riemann integrable} over \([a, b]\) and \(\int_a^b F(y) \, dy := I\).

The Riemann integral for multifunction with compact convex values was investigated by A. Dinghas [3] and M. Hukuhara [4]. Some properties of Riemann integral of multifunctions with convex closed bounded values may be found in paper [8].

2. Hadamard inequality in case of multifunctions

We believe that the following theorem is known. Nevertheless we prove it for convenience of the reader.

**Theorem 1**

Every \(*\)-concave multifunction \(F : [a, b] \to clb(X)\) is continuous on \((a, b)\) with respect to the Hausdorff metric.

**Proof.** Since all values of \(F\) are bounded we may find a constant \(M > 0\) such that \(\|\lambda F(a) + (1 - \lambda) F(b)\| \leq M\) for \(\lambda \in [0, 1]\). Thus by \(*\)-concavity of \(F\) we have \(\|F(x)\| \leq M, x \in [a, b]\).

Let us fix \(x_0 \in (a, b)\) and let \(x\) be a point belonging to the interval \((x_0, b)\). There exist \(\lambda, \mu \in (0, 1)\) such that \(x = \lambda x_0 + (1 - \lambda)b\) and \(x_0 = \mu x + (1 - \mu)a\). Hence \(\lambda = \frac{x - b}{x_0 - b} \to 1^-\) and \(\mu = \frac{x_0 - a}{x - a} \to 1^-\) as \(x \to x_0^+\). Then by the \(*\)-concavity we obtain

\[
e(F(x), F(x_0)) \leq e(\lambda F(x_0) + (1 - \lambda) F(b), F(x_0))
\]

\[
\leq d(\lambda F(x_0) + (1 - \lambda) F(b), \lambda F(x_0) + (1 - \lambda) F(x_0))
\]

\[
= (1 - \lambda)d(F(b), F(x_0))
\]

and

\[
e(F(x_0), F(x)) \leq e(\mu F(x) + (1 - \mu) F(a), F(x))
\]

\[
\leq d(\mu F(x) + (1 - \mu) F(a), \mu F(x) + (1 - \mu) F(x))
\]

\[
= (1 - \mu)d(F(a), F(x))
\]

\[
\leq 2M(1 - \mu),
\]

whence \(d(F(x), F(x_0)) \to 0\) as \(x \to x_0^+\). We have shown that \(F\) is right-hand side continuous at \(x_0\). The similar argument can be used to get the left-hand side continuity of \(F\) at \(x_0\).
**Remark 3**

A *-concave multifunction on \([a, b]\) need not be continuous. To see this it is enough to take \(F : [0, 1] \rightarrow clb(\mathbb{R})\) defined by

\[
F(x) = \begin{cases} 
\{0\}, & x > 0, \\
[0, 1], & x = 0.
\end{cases}
\]

The continuity of convex multifunctions can be obtained from Theorem 3.7 in [7]. We give here an independent, straightforward proof similar to that of Theorem 1.

**Theorem 1′**

Every convex multifunction \(F : [a, b] \rightarrow clb(X)\) is continuous on \((a, b)\) with respect to the Hausdorff metric and bounded on \([a, b]\).

**Proof.** At first we will prove that \(F\) is bounded on \([a, b]\). We observe that for every \(x \in [a, \frac{a+b}{2}]\) there exists \(\lambda \in \left[\frac{b}{2}, 1\right]\) such that \(\lambda x + (1 - \lambda)b = \frac{a+b}{2}\). Let us fix \(u \in F(b)\). The convexity of \(F\) yields

\[
\lambda F(x) + (1 - \lambda)u \subset F\left(\frac{a+b}{2}\right),
\]

whence

\[
F(x) \subset \frac{1}{\lambda} F\left(\frac{a+b}{2}\right) - \left(\frac{1}{\lambda} - 1\right)u.
\]

Thus \(F\) is bounded on \([a, \frac{a+b}{2}]\). In similar manner we show that \(F\) is bounded on \([\frac{a+b}{2}, b]\). Consequently there is a constant \(M\) such that \(||F(x)|| \leq M\) for \(x \in [a, b]\).

Let us fix \(x_0\) belonging to \((a, b)\) and let \(x_0 < x < b\). We can find \(\lambda, \mu \in (0, 1)\) such that \(x = \lambda x_0 + (1 - \lambda)b, x_0 = \mu x + (1 - \mu)a\). Clearly \(\lambda F(x_0) + (1 - \lambda)F(b) \subset F(x)\) and \(\mu F(x) + (1 - \mu)F(a) \subset F(x_0)\). We note that \(\lambda, \mu \to 1^{-}\) as \(x \to x_0^{+}\). By the convexity of \(F\) and properties of \(e\) we obtain two inequalities

\[
\begin{align*}
\lambda e(F(x_0), F(x)) &= e(\lambda F(x_0), \lambda F(x)) \\
&= e(\lambda F(x_0) + (1 - \lambda)F(b), \lambda F(x) + (1 - \lambda)F(b)) \\
&\leq e(F(x), \lambda F(x) + (1 - \lambda)F(b)) \\
&\leq d(F(x), \lambda F(x) + (1 - \lambda)F(b)) \\
&= (1 - \lambda)d(F(x), F(b)) \\
&\leq 2M(1 - \lambda),
\end{align*}
\]

\[
\begin{align*}
e(F(x), F(x_0)) &= \sup_{v \in F(x)} \rho(v, F(x_0)) \\
&\leq \sup_{v \in F(x)} \rho(v, \mu F(x) + (1 - \mu)F(a))
\end{align*}
\]
\[ e(F(x), \mu F(x) + (1 - \mu)F(a)) \leq d(F(x), \mu F(x) + (1 - \mu)F(a)) \]
\[ (1 - \mu)d(F(x), F(a)) \leq 2M(1 - \mu). \]

Consequently
\[ \lim_{x \to x_0^+} d(F(x), F(x_0)) = 0. \]

The left continuity of \( F \) at \( x_0 \) may be shown analogously.

A continuous multifunction \( F: [a, b] \rightarrow clb(X) \) is Riemann integrable on \([a, b]\) (cf. [8]). A \(*\)-concave multifunction on \([a, b]\) is commonly bounded on this interval. Therefore it is not difficult to see that a \(*\)-concave multifunction has to be Riemann integrable on each \([c, d]\) \(\subset [a, b]\) (cf. [8]).

In the case of convex functions on \([a, b]\) the following Hadamard inequality
\[ f\left( \frac{a + b}{2} \right) \leq \frac{1}{b - a} \int_a^b f(x) \, dx \leq \frac{f(a) + f(b)}{2} \]
is well known (cf. [5, pp. 196-197]). We are going to deal with suitable inclusion for convex and \(*\)-concave multifunctions.

**Theorem 2**
If \( F: [a, b] \rightarrow clb(X) \) is \(*\)-concave multifunction, then
\[
F\left( \frac{x + y}{2} \right) \subseteq \frac{1}{y - x} \int_x^y F(t) \, dt \subseteq \frac{F(x) + * F(y)}{2} \tag{1}
\]
for each \( x, y \) such that \( x < y \) and \([x, y]\) \( \subset [a, b]\).

**Proof.** Let us fix \( n \in \mathbb{N} \) and let \( x_i = x + iy - x \]
\[ n \] and \( \tau_i = x + \frac{2i - 1}{2n} (y - x) \) for \( i \in \{1, \ldots, n\} \).
These points create the partition \( \Delta_n = \{ x, x_1, \ldots, x_{n-1}, y \} \) of the interval \([x, y]\) and \( \tau = (\tau_1, \ldots, \tau_n) \) is a system of intermediate points. We note that
\[ \tau_i = \frac{x_{i-1} + x_i}{2} = \frac{2n - (2i - 1)}{2n} x + (2i - 1)y. \]

Using the \(*\)-concavity of \( F \) we obtain
\[ F(\tau_i) \subseteq \frac{2n - (2i - 1)}{2n} F(x) + \frac{2i - 1}{2n} F(y) \]
for \( i \in \{1, \ldots, n\} \). Summing up over \( i \) we get
\[ F(\tau_1) + \ldots + F(\tau_n) \]
\[ \subseteq \left( \frac{2n - 1}{2n} + \frac{2n - 3}{2n} + \cdots + \frac{1}{2n} \right) F(x) + \left( \frac{1}{2n} + \frac{3}{2n} + \cdots + \frac{2n - 1}{2n} \right) F(y). \]
Since $1 + 3 + \cdots + (2n - 1) = n^2$, we obtain
\[
\frac{1}{y-x} \left[ F(\tau_1) + \cdots + F(\tau_n) \right] \frac{y-x}{n} \subset \frac{F(x) + F(y)}{2}.
\] (2)

Now we let $n \to \infty$. Then $\delta(\Delta_n) \to 0$ and with respect to the definition of the integral, by (2) and by the closedness of the set $\frac{1}{2}(F(x) + F(y))$ we have
\[
\frac{1}{y-x} \int_x^y F(t) \, dt \subset \frac{F(x) + F(y)}{2}.
\]

To obtain the first inclusion of (1) we take an even positive integer $n$. Let $k = n/2$ and let us choose $i \in \{1, \ldots, k\}$. We note that $\frac{1}{2}(\tau_i + \tau_j) = \frac{1}{2}(x + y)$ for $j = n + 1 - i$. Again by the *-concavity of $F$ we infer
\[
F \left( \frac{x+y}{2} \right) \subset \frac{1}{2}(F(\tau_1) + F(\tau_j)).
\]
Summing up over $i \in \{1, \ldots, k\}$ leads to
\[
kF \left( \frac{x+y}{2} \right) \subset k \left[ F(\tau_1) + \cdots + F(\tau_k) + F(\tau_{k+1}) + \cdots + F(\tau_n) \right]
\]
or
\[
F \left( \frac{x+y}{2} \right) \subset \frac{1}{y-x} \left[ F(\tau_1) + \cdots + F(\tau_n) \right] \cdot \frac{y-x}{n}
\] (3)
for all even $n$. The right-hand side of inclusion (3) tends to $\frac{1}{y-x} \int_x^y F(t) \, dt$ as $n \to \infty$. Hence
\[
F \left( \frac{x+y}{2} \right) \subset \frac{1}{y-x} \int_x^y F(t) \, dt.
\]

The proof of the next theorem runs similarly.

**Theorem 2'**

If $F: [a, b] \to clb(X)$ is a convex multifunction, then the following inclusions hold
\[
\frac{F(x) + F(y)}{2} \subset \frac{1}{y-x} \int_x^y F(t) \, dt \subset F \left( \frac{x+y}{2} \right)
\] (4)
for all intervals $[x, y] \subset [a, b]$.

Inclusions (4) for the Aumann integral may be found in the paper of E. Sadrowska [9, Theorem 1], where the integral Jensen inequality is applied (see the paper of J. Matkowski and K. Nikodem [6]). The assumptions of Theorem 2' differ somewhat from that of Theorem 1 in [9].
3. Hardy-Littlewood-Pólya majorization theorem for multifunctions

In this part of the note we are going to transfer the Hardy–Littlewood–Pólya majorization principle for convex functions (cf. [5, ch. 8, § 5]) to convex and \(*\)-concave multifunctions.

**Theorem 3**

Let \(x_1, x_2, y_1, y_2\) be real numbers such that \(x_2 \leq x_1, y_2 \leq y_1, x_1 \leq y_1, x_1 + x_2 = y_1 + y_2\). If \(F: \mathbb{R} \rightarrow clb(X)\) is \(*\)-concave, then

\[
F(x_1) + F(x_2) \subset F(y_1) + F(y_2).
\] (5)

**Proof.** The assumptions of the theorem imply the inequality \(y_2 \leq x_2 \leq x_1 \leq y_1\). At first we assume that \(y_1 \neq y_2\). Setting \(\lambda = \frac{y_1 - x_2}{y_1 - y_2}\), \(\mu = \frac{y_1 - x_1}{y_1 - y_2}\) by the \(*\)-concavity we have

\[
F(x_2) = F(\lambda y_2 + (1 - \lambda)y_1) \subset \lambda F(y_2) + (1 - \lambda)F(y_1),
\]

\[
F(x_1) = F(\mu y_2 + (1 - \mu)y_1) \subset \mu F(y_2) + (1 - \mu)F(y_1).
\]

Multiplying the above inclusions by \(y_1 - y_2\) and summing them up together we obtain

\[
(y_1 - y_2)(F(x_1) + F(x_2)) \subset (y_2 - y_2 + x_1 - y_2)F(y_1) + (y_1 - x_2 + y_1 - x_1)F(y_2).
\]

The equality \(x_1 + x_2 = y_1 + y_2\) and the above inclusions lead to

\[
F(x_1) + F(x_2) \subset F(y_1) + F(y_2).
\]

If \(y_1 = y_2\), then \(y_1 = x_1 = x_2 = y_2\) and condition (5) holds true.

Theorem 3 for concave multifunctions can be found in [7, Theorem 2.14] in another formulation. The same concerns the next theorem. Its proof is similar to the previous one.

**Theorem 3’**

Let \(x_1, x_2, y_1, y_2\) be real numbers such that \(x_2 \leq x_1, y_2 \leq y_1, x_1 \leq y_1\) and, \(x_1 + x_2 = y_1 + y_2\). If \(F: \mathbb{R} \rightarrow clb(X)\) is convex, then

\[
F(y_1) + F(y_2) \subset F(x_1) + F(x_2).
\]

**Corollary 1**

Let \(a, b, c\) be non-negative numbers and let \(a + b \leq c\). If \(F: [0, \infty) \rightarrow clb(X)\) is a \(*\)-concave multifunction, then

\[
F(a + b) + F(c) \subset F(a) + F(b + c).
\]
Proof. To obtain the Corollary from Theorem 3 it is enough to set \( x_1 = c, \)
\( x_2 = a + b, \)
\( y_1 = b + c, \)
\( y_2 = a \) (see \([5, \text{pp. 194-195}])\).

**Corollary 2**

Let \( x_1, x_2, y_1, y_2 \) be real numbers satisfying the conditions: \( x_2 \leq x_1, \)
\( y_2 \leq y_1, \)
\( x_1 \leq y_1 \) and \( x_1 + x_2 \leq y_1 + y_2. \) If \( F: \mathbb{R} \rightarrow clb(X) \) is an increasing \(*\)-concave multifunction, then

\[
F(x_1) + F(x_2) \subset F(y_1) + F(y_2)
\]

holds true.

**Proof.** Taking \( z_1 = y_1 \) and \( z_2 = x_1 + x_2 - y_1 \) we can easily check that the numbers \( x_1, x_2, z_1, z_2 \) satisfy the assumption of Theorem 3. Hence

\[
F(x_1) + F(x_2) \subset F(z_1) + F(z_2).
\]

Moreover, \( F \) is increasing and \( z_2 \leq y_2, \) so

\[
F(x_1) + F(x_2) \subset F(y_1) + F(y_2).
\]

**Theorem 4**

Assume that \( x_i, y_i, i \in \{1, \ldots, n\} \) are real numbers such that

\[
x_n \leq x_{n-1} \leq \ldots \leq x_1, \quad y_n \leq y_{n-1} \leq \ldots \leq y_1,
\]

\[
\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i, \quad k \in \{1, \ldots, n-1\}, \quad \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i
\]

and

\[
x_{k+1} \leq y_k, \quad k \in \{2, \ldots, n-1\}.
\]

If \( F: \mathbb{R} \rightarrow clb(X) \) is a \(*\)-concave multifunction, then

\[
F(x_1) + \cdots + F(x_n) \subset F(y_1) + \cdots + F(y_n).
\]

**Proof.** The theorem is valid for \( n = 2 \) thanks to Theorem 3. Now we assume (9) true for an \( n \in \mathbb{N}, n \geq 2 \) and take arbitrary numbers \( x_i, y_i, i \in \{1, \ldots, n, n + 1\} \) satisfying

\[
x_{n+1} \leq x_n \leq \ldots \leq x_1, \quad y_{n+1} \leq y_n \leq \ldots \leq y_1,
\]

\[
\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i, \quad k \in \{1, \ldots, n\}, \quad \sum_{i=1}^{n+1} x_i = \sum_{i=1}^{n+1} y_i
\]

and

\[
x_{k+1} \leq y_k, \quad k \in \{2, \ldots, n\}.
\]
By (7\(_{n+1}\)) we have

\[
\sum_{i=1}^{n} x_i = \sum_{i=1}^{n-1} y_i + (y_n + y_{n+1} - x_{n+1}).
\]

According to the induction hypothesis

\[
F(x_1) + \cdots + F(x_n) \subset F(y_1) + \cdots + F(y_{n-1}) + F(y_n + y_{n+1} - x_{n+1})
\]

since \(y_n + y_{n+1} - x_{n+1} \leq y_{n-1}\) (see (6\(_{n+1}\)) and (7\(_{n+1}\))). If we show that

\[
F(y_n + y_{n+1} - x_{n+1}) + F(x_{n+1}) \subset F(y_n) + F(y_{n+1})
\]

(10)
holds, the proof will be complete.

Consider two cases: (a) \(x_{n+1} \leq y_n + y_{n+1} - x_{n+1}\) and (b) \(x_{n+1} > y_n + y_{n+1} - x_{n+1}\). In case (b) \((y_n + y_{n+1} - x_{n+1}) + x_{n+1} = y_n + y_{n+1}, y_{n+1} \leq y_n, y_n + y_{n+1} - x_{n+1} < x_{n+1}\) and \(x_{n+1} \leq y_n\) according to (8\(_{n+1}\)). By Theorem 3 condition (10) holds. In case (a), \(x_{n+1} + (y_n + y_{n+1} - x_{n+1}) = y_n + y_{n+1}, y_{n+1} \leq y_n, x_{n+1} \leq y_n + y_{n+1} - x_{n+1}\) and \(y_n + y_{n+1} - x_{n+1} = y_n + (y_{n+1} - x_{n+1}) \leq y_n\) because \(y_{n+1} \leq x_{n+1}\). By Theorem 3 condition (10) holds.

**Theorem 4’**

Assume that \(x_i, y_i, i \in \{1, \ldots, n\}\) are real numbers such that

\[
x_n \leq x_{n-1} \leq \cdots \leq x_1, \quad y_n \leq y_{n-1} \leq \cdots \leq y_1,
\]

\[
\sum_{i=1}^{k} x_i \leq \sum_{i=1}^{k} y_i, \quad k \in \{1, \ldots, n-1\}, \quad \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i
\]

and

\[
x_{k+1} \leq y_k, \quad k \in \{2, \ldots, n-1\}.
\]

If \(F : \mathbb{R} \rightarrow \text{clb}(X)\) is a convex multifunction, then

\[
F(y_1) + \cdots + F(y_n) \subset F(x_1) + \cdots + F(x_n).
\]

Results of the same kind as Theorem 4 and 4’, formulated in some other language, were obtained by K. Nikodem (cf. [7, Theorem 2.14]).

**References**


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