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Boundary value problem for the second order impulsive delay differential equations

Abstract. We present some existence and uniqueness result for a boundary value problem for functional differential equations of second order with impulses at fixed points.

1. Introduction

Impulsive differential equations describe processes that are subjected to abrupt changes in their state at fixed or variable times and present a natural framework for mathematical modelling of several real-world problems (see [6, 10]). In consequence, the study of impulsive differential equations is of great interest both for the theoretical and practical point of view.

The theory of impulsive differential equations and impulsive functional differential equations has been an important area of investigations in recent years. Among others the existence of solutions of the first and the second order impulsive functional differential equations by using the fixed point argument such as the Banach contraction principle, fixed point index theory and monotone iterative technique were discussed. We mention here the papers [1, 2, 3, 4, 5, 7, 8, 9, 12] and the references therein.

In the present paper we shall investigate the existence of the solutions of the boundary value problem for the second order delay differential systems with impulses at fixed points. The existence results for the boundary value problem for the second order delay differential equations of the above type without impulsive conditions have been studied in [11].

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2. Preliminaries

In this paper we consider the following second order boundary value problem with impulses at fixed points

$$\begin{aligned} x''(t) &= f(t, x_t), && t \in J' = J \setminus \{t_1, \dots, t_p\}, \\ \triangle x(t_k) &= I_{0k}(x(t_k), x'(t_k)), && k = 1, \dots, p, \\ \triangle x'(t_k) &= I_{1k}(x(t_k), x'(t_k)), && k = 1, \dots, p, \\ x_0 &= \phi, \\ x'(T) &= \beta x'(0), && \beta > 1, \end{aligned} \tag{1}$$

where $J = [0, T]$, $T > 0$, $0 = t_0 < t_1 < \dots < t_p < t_{p+1} = T$, $f \colon J \times PC([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^n$ is given a function, $\phi \in PC([-\tau, 0], \mathbb{R}^n)$, $\tau > 0$,

$$\begin{aligned} PC([-\tau,0],\mathbb{R}^n) = \{x \colon [-\tau,0] \to \mathbb{R}^n :\ & x(t^-) = x(t) \text{ for all } t \in (-\tau, 0],\ x(t^+) \\ & \text{exists for all } t \in [-\tau, 0),\ \text{and } x(t^+) = x(t) \text{ for all} \\ & \text{but at most a finite number of points } t \in [-\tau, 0)\}. \end{aligned}$$

For any function $x \colon [-\tau, T] \to \mathbb{R}^n$ and any $t \in J$, we let x_t denote the function $x_t \colon [-\tau, 0] \to \mathbb{R}^n$ defined by

$$x_t(s) = x(t+s), \qquad s \in [-\tau, 0].$$

Here $x_t(\cdot)$ represents the history of the state from time $t - \tau$, up to the present time t. Condition $x_0 = \phi$ implies that $x(s) = \phi(s)$, $s \in [-\tau, 0]$. The supremum norm of $\phi \in PC([-\tau, 0], \mathbb{R}^n)$ is defined by

$$\|\phi\|_0 = \sup_{-\tau \le s \le 0} \|\phi(s)\|.$$

Let $L^1([-\tau, 0], \mathbb{R}^n)$ denote the Banach space of Lebesque integrable functions $y \colon [-\tau, 0] \to \mathbb{R}^n$ with norm

$$\|y\|_{L^1} = \int_{-\tau}^{0} \|y(t)\|\, dt.$$

Obviously $PC([-\tau, 0], \mathbb{R}^n) \subset L^1([-\tau, 0], \mathbb{R}^n)$ and for $\phi \in PC([-\tau, 0], \mathbb{R}^n)$,

$$\|\phi\|_{L^1} \le \tau \|\phi\|_0.$$

$\triangle x(t_k)$, $\triangle x'(t_k)$ denote the jump of $x(t)$, $x'(t)$ at $t = t_k$, i.e.

$$\begin{aligned} \triangle x(t_k) &= x(t_k^+) - x(t_k^-), \\ \triangle x'(t_k) &= x'(t_k^+) - x'(t_k^-), \end{aligned}$$

where $x(t_k^+)$, $x'(t_k^+)$, $x(t_k^-)$, $x'(t_k^-)$ represent the right and left limits of $x(t)$, $x'(t)$ at $t = t_k$, respectively, $I_{0k}, I_{1k} \colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$.

In order to define the concept of solution for (1) we introduce the following sets of functions

$$PC[J,\mathbb{R}^n] = \{x\colon J \to \mathbb{R}^n :\ x \text{ is continuous at } t \neq t_k,\ \text{left continuous at } t = t_k, \\ \text{and } x(t_k^+) \text{ exists, } k = 1,\dots,p\},$$

$$PC^1[J,\mathbb{R}^n] = \{x \in PC[J,\mathbb{R}^n] :\ x'(t) \text{ exists and is continuous at } t \neq t_k, \\ \text{and } x'(t_k^+),\ x'(t_k^-) \text{ exist for } k = 1,\dots,p\}.$$

We define $x'(t_k) = x'(t_k^-)$. Moreover, in (1) and in what follows, $x'(t_k)$ is understood as $x'(t_k^-)$. Note that for $x \in PC^1[J,\mathbb{R}^n]$, $x' \in PC[J,\mathbb{R}^n]$ and $PC[J,\mathbb{R}^n]$, $PC^1[J,\mathbb{R}^n]$ are Banach spaces with the norms

$$\|x\|_{PC} = \max\{e^{-t}\max\{\|x(s)\| :\ s \in [0,t]\},\, t \in J\},$$
$$\|x\|_{PC^1} = \max\{\|x\|_{PC}, \|x'\|_{PC}\}.$$

We shall prove an existence result for (1) by using the Banach contraction principle.

3. Auxiliary result

Let us start by defining what we mean by a solution of problem (1). Denote $C^\star = PC^1([-\tau,T],\mathbb{R}^n) \cap C^2(J',\mathbb{R}^n)$.

Definition 3.1
A function $x \in C^\star$ is said to be a solution of (1) if x satisfies (1).

We need the following auxiliary lemma.

Lemma 3.2
Assume that $f \in C(J \times L^1([-\tau,0],\mathbb{R}^n),\mathbb{R}^n)$. Function $x \in C^\star$ is a solution of (1) *if and only if x is a solution of the following integral equation*

$$x(t) = \begin{cases} \phi(t), \quad t \in [-\tau,0], \\ \phi(0) + \dfrac{t}{\beta-1}\Big[\displaystyle\int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k))\Big] \\ \qquad + \displaystyle\int_0^t (t-s)f(s,x_s)\,ds \\ \qquad + \displaystyle\sum_{0<t_k<t} (I_{0k}(x(t_k),x'(t_k)) + (t-t_k)I_{1k}(x(t_k),x'(t_k))), \quad t \in J. \end{cases}$$

Proof. First we prove that the integrals $\int_0^T f(s,x_s)\,ds$, $\int_0^t (t-s)f(s,x_s)\,ds$ exist. Consider the function $x \in PC^1(J,\mathbb{R}^n)$ such that

$$x_t(s) = x(t+s), \qquad s \in [-\tau,0]$$

and

$$x(t+s)=\phi(t+s), \qquad \text{if } t+s\le 0.$$

We have

$$x_t\in PC([-\tau,0],\mathbb{R}^n)\subseteq L^1([-\tau,0],\mathbb{R}^n).$$

For any $t_0\in J$, if $t\to t_0$ then

$$x_t(s)\to x_{t_0}(s) \qquad \text{a.e. } s\in[-\tau,0]$$

and

$$\lim_{t\to t_0}\|x_t-x_{t_0}\|_{L^1}=\lim_{t\to t_0}\int_{-\tau}^{0}\|x(t+s)-x(t_0+s)\|\,ds=0.$$

This implies that for any $x\in PC^1(J,\mathbb{R}^n)$, $f(t,x_t)$ is continuous on J except on a set of countable points. Then $f(t,x_t)$ is Lebesque integrable on any bounded interval.

Assume that the function $x\in C^\star$ is a solution of (1). The function x can be written of the form

$$\begin{aligned} x(t)=x(0)+tx'(0)+\int_0^t(t-s)x''(s)\,ds\\ +\sum_{0<t_k<t}[(x(t_k^+)-x(t_k))+(t-t_k)(x'(t_k^+)-x'(t_k))]. \end{aligned} \tag{2}$$

Differentiating (2), we get

$$x'(t)=x'(0)+\int_0^t x''(s)\,ds+\sum_{0<t_k<t}(x'(t_k^+)-x'(t_k)).$$

Hence

$$x'(T)=x'(0)+\int_0^T x''(s)\,ds+\sum_{k=1}^{p}(x'(t_k^+)-x'(t_k)).$$

Using the boundary condition we obtain

$$x'(0)+\int_0^T x''(s)\,ds+\sum_{k=1}^{p}(x'(t_k^+)-x'(t_k))=\beta x'(0).$$

Thus

$$x'(0)=\frac{1}{\beta-1}\Big[\int_0^T x''(s)\,ds+\sum_{k=1}^{p}(x'(t_k^+)-x'(t_k))\Big]. \tag{3}$$

Equation (2), together with (1) and (3) implies

$$\begin{aligned} x(t)=\Phi(0)+\frac{t}{\beta-1}\Big[\int_0^T f(s,x_s)\,ds+\sum_{k=1}^{p}I_{1k}(x(t_k),x'(t_k))\Big]\\ +\int_0^t(t-s)f(s,x_s)\,ds\\ +\sum_{0<t_k<t}(I_{0k}(x(t_k),x'(t_k))+(t-t_k)I_{1k}(x(t_k),x'(t_k))). \end{aligned} \tag{4}$$

Conversely, if x is a solution of equation (4), then direct differentiation of (4) gives

$$\begin{aligned} x'(t) = \frac{1}{\beta-1}\Big[\int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k))\Big] \\ + \int_0^t f(s,x_s)\,ds + \sum_{0<t_k<t} I_{1k}(x(t_k),x'(t_k)), \qquad t\in J' \end{aligned} \tag{5}$$

and

$$x''(t) = f(t,x_t), \qquad t\in J'.$$

Obviously

$$\begin{aligned} \triangle x(t_k) &= I_{0k}(x(t_k),x'(t_k)), \qquad k=1,\dots,p,\\ \triangle x'(t_k) &= I_{1k}(x(t_k),x'(t_k)), \qquad k=1,\dots,p. \end{aligned}$$

From (5) we have

$$x'(0) = \frac{1}{\beta-1}\Big[\int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k))\Big]$$

and

$$\begin{aligned} x'(T) &= \frac{1}{\beta-1}\Big[\int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k))\Big] \\ &\quad + \int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k)) \\ &= \frac{\beta}{\beta-1}\Big[\int_0^T f(s,x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k),x'(t_k))\Big], \end{aligned}$$

which gives

$$x'(T) = \beta x'(0).$$

4. Main result

We introduce the following assumptions on the functions appearing in the problem (1):

(H1) There exists a function $m\in C(J,\mathbb{R}^+)$ such that

$$\|f(t,x_t)-f(t,y_t)\| \le m(t)e^{-t}\|x_t-y_t\|_{L^1}$$

for any $t\in J$ and $x,y\in PC^1(J,\mathbb{R}^n)$.

(H2) There exist nonnegative constants c_{ik}, $\tilde c_{ik}$, $i=0,1$, $k=1,2,\dots,p$, such that the functions I_{ik}, $i=0,1$, $k=1,2,\dots,p$ verify the following conditions

$$\begin{aligned} &\|I_{ik}(x(t_k),x'(t_k)) - I_{ik}(y(t_k),y'(t_k))\| \\ &\qquad\le c_{ik}e^{-t_k}\|x(t_k)-y(t_k)\| + \tilde c_{ik}e^{-t_k}\|x'(t_k)-y'(t_k)\| \end{aligned}$$

for any $x,y\in PC^1(J,\mathbb{R}^n)$.

Denote

$$M = \int_0^T m(r)\,dr, \qquad C_0 = \sum_{k=1}^{p} (c_{0k} + \tilde{c}_{0k}), \qquad C_1 = \sum_{k=1}^{p} (c_{1k} + \tilde{c}_{1k}).$$

Theorem 4.1
Assume that $f \in C(J \times L^1([-\tau, 0], \mathbb{R}^n), \mathbb{R}^n)$. *If the assumptions* $(H1)$, $(H2)$ *hold and*

$$\frac{\beta}{\beta - 1}(\tau M + C_1) + C_0 < 1$$

then the problem (1) *has a unique solution* $x \in C^\star$.

Proof. We transform the problem (1) into a fixed point problem. For $x \in PC^1(J, \mathbb{R}^n)$, let

$$\begin{aligned}(Ax)(t) = \phi(0) + \frac{t}{\beta - 1}\Big[\int_0^T f(s, x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k), x'(t_k))\Big] \\ + \int_0^t (t - s) f(s, x_s)\,ds \\ + \sum_{0<t_k<t} (I_{0k}(x(t_k), x'(t_k)) + (t - t_k) I_{1k}(x(t_k), x'(t_k))),\end{aligned} \tag{6}$$

where $t \in J$, $x_s(r) = x(s + r) = \phi(s + r)$ for $s + r \le 0$. Differentiation of (6) gives

$$\begin{aligned}(Ax)'(t) = \frac{1}{\beta - 1}\Big[\int_0^T f(s, x_s)\,ds + \sum_{k=1}^{p} I_{1k}(x(t_k), x'(t_k))\Big] \\ + \int_0^t f(s, x_s)\,ds + \sum_{0<t_k<t} I_{1k}(x(t_k), x'(t_k)), \qquad t \in J.\end{aligned}$$

For $x, y \in PC^1(J, \mathbb{R}^n)$ we have

$$\begin{aligned}\|(Ax)(t) - (Ay)(t)\| \le \frac{\beta t}{\beta - 1}\Big[\int_0^T \|f(s, x_s) - f(s, y_s)\|\,ds \\ + \sum_{k=1}^{p} \|I_{1k}(x(t_k), x'(t_k)) - I_{1k}(y(t_k), y'(t_k))\|\Big] \\ + \sum_{0<t_k<t} \|I_{0k}(x(t_k), x'(t_k)) - I_{0k}(y(t_k), y'(t_k))\|\end{aligned}$$

and

$$\begin{aligned}\|(Ax)'(t) - (Ay)'(t)\| \le \frac{\beta}{\beta - 1}\Big[\int_0^T \|f(s, x_s) - f(s, y_s)\|\,ds \\ + \sum_{k=1}^{p} \|I_{1k}(x(t_k), x'(t_k)) - I_{1k}(y(t_k), y'(t_k))\|\Big],\end{aligned}$$

for $t \in J$.

This, together with the assumptions (H1), (H2) gives

$$\begin{aligned}\|(Ax)(t)-(Ay)(t)\| \le \frac{\beta t}{\beta-1}\Big[\int_0^T \tau m(s)e^{-s}\|x_s-y_s\|_0\,ds\\ +\sum_{k=1}^{p} e^{-t_k}(c_{1k}\|x(t_k)-y(t_k)\|+\tilde c_{1k}\|x'(t_k)-y'(t_k)\|)\Big]\\ +\sum_{0<t_k<t} e^{-t_k}(c_{0k}\|x(t_k)-y(t_k)\|+\tilde c_{0k}\|x'(t_k)-y'(t_k)\|)\end{aligned}$$

and

$$\begin{aligned}\|(Ax)'(t)-(Ay)'(t)\| \le \frac{\beta}{\beta-1}\Big[\int_0^T \tau m(s)e^{-s}\|x_s-y_s\|_0\,ds\\ +\sum_{k=1}^{p} e^{-t_k}(c_{1k}\|x(t_k)-y(t_k)\|+\tilde c_{1k}\|x'(t_k)-y'(t_k)\|)\Big].\end{aligned}$$

Notice that if $s\in[0,\tau]$, then

$$\begin{aligned}\|x_s-y_s\|_0 &= \sup_{r\in[-\tau,0]}\|x(s+r)-y(s+r)\|\\ &=\max\{\|x(s+r)-y(s+r)\|,\ r\in[-s,0]\}\\ &=\max\{\|x(r)-y(r)\|,\ r\in[0,s]\}.\end{aligned}$$

If $s\in(\tau,T]$,

$$\begin{aligned}\|x_s-y_s\|_0 &= \sup_{r\in[-\tau,0]}\|x(s+r)-y(s+r)\|\\ &=\max\{\|x(r)-y(r)\|,\ r\in[s-\tau,s]\}\\ &\le\max\{\|x(r)-y(r)\|,\ r\in[0,s]\}.\end{aligned}$$

Therefore

$$\begin{aligned}&\|(Ax)(t)-(Ay)(t)\|\\ &\quad\le \frac{\beta t}{\beta-1}\Big[\int_0^T \tau m(s)e^{-s}\max_{r\in[0,s]}\|x(r)-y(r)\|\,ds\\ &\quad+\sum_{k=1}^{p} e^{-t_k}\Big(c_{1k}\max_{r\in[0,t_k]}\|x(r)-y(r)\|+\tilde c_{1k}\max_{r\in[0,t_k]}\|x'(r)-y'(r)\|\Big)\Big]\\ &\quad+\sum_{0<t_k<t} e^{-t_k}\Big(c_{0k}\max_{r\in[0,t_k]}\|x(r)-y(r)\|+\tilde c_{0k}\max_{r\in[0,t_k]}\|x'(r)-y'(r)\|\Big)\end{aligned}$$

and

$$\begin{aligned}&\|(Ax)'(t)-(Ay)'(t)\|\\ &\quad\le \frac{\beta}{\beta-1}\Big[\int_0^T \tau m(s)e^{-s}\max_{r\in[0,s]}\|x(r)-y(r)\|\,ds\\ &\quad+\sum_{k=1}^{p} e^{-t_k}\Big(c_{1k}\max_{r\in[0,t_k]}\|x(r)-y(r)\|+\tilde c_{1k}\max_{r\in[0,t_k]}\|x'(r)-y'(r)\|\Big)\Big]\end{aligned}$$

for $t\in J$.

This implies that

$$\|(Ax)(t)-(Ay)(t)\| \le \frac{\beta t}{\beta-1}(\tau M + C_1)\|x-y\|_{PC^1} + C_0\|x-y\|_{PC^1}$$

and

$$\|(Ax)'(t)-(Ay)'(t)\| \le \frac{\beta}{\beta-1}(\tau M + C_1)\|x-y\|_{PC^1}, \qquad t\in J.$$

In consequence

$$\max_{s\in[0,t]}\|(Ax)(s)-(Ay)(s)\| \le \Big(\frac{\beta t}{\beta-1}(\tau M + C_1)+C_0\Big)\|x-y\|_{PC^1}$$

and

$$\max_{s\in[0,t]}\|(Ax)'(s)-(Ay)'(s)\| \le \frac{\beta}{\beta-1}(\tau M + C_1)\|x-y\|_{PC^1}, \qquad t\in J.$$

Then

$$e^{-t}\max_{s\in[0,t]}\|(Ax)(s)-(Ay)(s)\| \le \Big(\frac{\beta}{\beta-1}(\tau M + C_1)+C_0\Big)\|x-y\|_{PC^1}$$

and

$$e^{-t}\max_{s\in[0,t]}\|(Ax)'(s)-(Ay)'(t)\| \le \frac{\beta}{\beta-1}(\tau M + C_1)\|x-y\|_{PC^1}, \qquad t\in J.$$

Hence we have the following estimate

$$\|Ax-Ay\|_{PC^1} \le \alpha\|x-y\|_{PC^1}$$

with $\alpha = \frac{\beta}{\beta-1}(\tau M + C_1) + C_0$. Thus A is a contractive operator and by Banach fixed point theorem, A has a unique fixed point $x \in PC^1(J,\mathbb{R}^n)$. The proof is complete.

When the impulsive functions are constants the boundary value problem (1) is of the form

$$\begin{aligned} x''(t) &= f(t,x_t), && t\in J',\\ \triangle x(t_k) &= \mu_{0k}, && k=1,\dots,p,\\ \triangle x'(t_k) &= \mu_{1k}, && k=1,\dots,p,\\ x_0 &= \phi, &&\\ x'(T) &= \beta x'(0), && \beta>1, \end{aligned} \tag{7}$$

with $\mu_{0k},\mu_{1k}\in\mathbb{R}^n$, $k=1,\dots,p$.

As a consequence of the previous theorem, we have the following result.

Corollary 4.2
Assume that $f\in C(J\times L^1([-\tau,0],\mathbb{R}^n),\mathbb{R}^n)$. If the assumption $(H1)$ holds and

$$\int_0^T m(r)\,dr < \frac{\beta-1}{\beta\tau}$$

then the problem (7) has a unique solution $u\in C^\star$.

When $\mu_{ik}=0$, $i=0,1$, $k=1,\dots,p$ we obtain existence result for the boundary problem for second order delay differential equation without impulses under different assumptions than in [11].

References

[1] A. Cabada, J.J. Nieto, D. Franco, S.I. Trofimchuk, *A generalization of the monotone method for second order periodic boundary value problem with impulses at fixed points*, Dynam. Contin. Discrete Impuls. Systems **7** (2000), no. 1, 145–158. Cited on 27.

[2] Wei Ding, Maoan Han, *Periodic boundary value problem for the second order impulsive functional differential equations*, Indian J. Pure Appl. Math. **35** (2004), no. 8, 949–968. Cited on 27.

[3] J.R. Graef, A. Ouahab, *Some existence and uniqueness results for impulsive functional differential equations with variable times in Fréchet spaces*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. **14** (2007), no. 1, 27–45. Cited on 27.

[4] Dajun Guo, *A class of second-order impulsive integro-differential equations on unbounded domain in a Banach space*, Appl. Math. Comput. **125** (2002), no. 1, 59–77. Cited on 27.

[5] Xilin Fu, Baoqiang Yan, *The global solutions of impulsive retarded functional-differential equations*, Int. J. Appl. Math. **2** (2000), no. 3, 389–398. Cited on 27.

[6] V. Lakshmikantham, D.D. Baĭnov, P.S. Simeonov, *Theory of impulsive differential equations*, Series in Modern Applied Mathematics, 6, World Scientific Publishing Co., Inc., Teaneck, NJ, 1989. Cited on 27.

[7] Jianli Li, Jianhua Shen, *Periodic boundary value problems for delay differential equations with impulses*, J. Comput. Appl. Math. **193** (2006), no. 2, 563–573. Cited on 27.

[8] Long Tu Li, Zhi Cheng Wang, Xiang Zheng Qian, *Boundary value problems for second order delay differential equations*, Appl. Math. Mech. (English Ed.) **14** (1993), no. 6, 573–580. Cited on 27.

[9] Bing Liu, *Positive solutions of second-order three-point boundary value problems with change of sign*, Comput. Math. Appl. **47** (2004), no. 8-9, 1351–1361. Cited on 27.

[10] A.M. Samoĭlenko, N.A. Perestyuk, *Impulsive differential equations*, World Scientific Series on Nonlinear Science, Series A: Monographs and Treatises, 14, World Scientific Publishing Co., Inc., River Edge, NJ, 1995. Cited on 27.

[11] L. Skóra, *Boundary value problems for second order delay differential equations*, Opuscula Math. **32** (2012), no. 3, 551–558. Cited on 27 and 34.

[12] Haihua Wang, Haibo Chen, *Boundary value problem for second-order impulsive functional differential equations*, Appl. Math. Comput. **191** (2007), no. 2, 582–591. Cited on 27.

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