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Comparison of CA method and neural networks as the reanalysis tools for identification of the modified load-bearing walls natural frequencies

Introduction

Every change in the geometry of structure (i.e. in the stiffness and mass of the structure) causes the changes in its dynamic properties, among them – in natural frequencies of vibrations. The computational effort required to resolve the new eigenproblem (reanalysis of the eigenproblem) using finite element method (FEM) [11, 12] can be significant, particularly in case of various modification variants and a lot of degrees of freedom (DOF). That is why application of reanalysis techniques for computation of natural frequencies of modified walls is discussed in this paper. Reanalysis methods allow to analyse modified structures using some information about the structure before modification. Thereby, reduction of the computational effort is possible.

That is why two reanalysis techniques for computation of natural frequencies of the modified typical medium-height load-bearing walls are discussed and compared in the paper: combined approximations hybrid method (CA) [5, 6, 7] and back-propagation neural networks (BPNNs) [1, 2, 4]. The small and the large changes of the wall stiffness and mass resulting from the new door openings size and position were analysed.

Outline of applied reanalysis approaches

CA method

Combined approximations (CA) hybrid method [6, 8] applied in case of computing the natural vibration frequencies of the modified structure enables us to strongly reduce the number of eigenproblem equations.

In this method, the information about the initial structure and introduced modifications is included in so-called basis vectors. The basis vectors (global approximations) are computed using the binomial series (local approximations).

The eigenproblem in case of the structure before and after modification is written by Equations (1) and (2), respectively:

$$\mathbf{K\Phi}_i = \lambda_i \mathbf{M\Phi}_i,\ i = 1,\ldots, p, \tag{1}$$

$$\mathbf{K}_{\mathrm{M}} \mathbf{\Phi}_{\mathrm{M}i} = \lambda_{\mathrm{M}i} \mathbf{M}_{\mathrm{M}} \mathbf{\Phi}_{\mathrm{M}i},\ i = 1,\ldots, p, \tag{2}$$

where: $\mathbf{K}$, $\mathbf{K}_{\mathrm{M}}$ – stiffness matrices; $\mathbf{M}$, $\mathbf{M}_{\mathrm{M}}$ – mass matrices; Φ_i, $\Phi_{\mathrm{M}i}$ – mode shapes; λ_i, $\lambda_{\mathrm{M}i}$ – eigenvalues before and after structure modification.

Taking into account the changes in stiffness ($\Delta\mathbf{K}$) and mass ($\Delta\mathbf{M}$) matrices (corresponding to the geometrical changes of the structure), the following relations can be taken into consideration:

$$\begin{aligned} \mathbf{K}_{\mathrm{M}} &= \mathbf{K} + (\Delta\mathbf{K}). \\ \mathbf{M}_{\mathrm{M}} &= \mathbf{M} + \Delta\mathbf{M} \end{aligned} \tag{3}$$

Computation of matrix $\mathbf{r}_{\mathrm{B}}$ which contains the basis vectors is the next step of CA method [5]:

$$\mathbf{r}_{\mathrm{B}} = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_s], \tag{4}$$

where: $\mathbf{r}_1,\ldots,\mathbf{r}_s$ – the basis vectors, s – the number of basis vectors (s is much smaller than the number of degrees of freedom).

The first basis vector $\mathbf{r}_1$ and each subsequent vector $\mathbf{r}_k$ are computed according the following formulas [7]:

$$\mathbf{r}_1 = \mathbf{K}^{-1} \mathbf{M}_{\mathrm{M}} \mathbf{\Phi}_i, \tag{5}$$

$$\mathbf{r}_k = -\mathbf{B}\mathbf{r}_{k\text{-}1},\ k = 2, 3, \ldots, s, \tag{6}$$

where: $\mathbf{B} = \mathbf{K}^{-1}\Delta\mathbf{K}$.

Reduced stiffness $\mathbf{K}_{\mathrm{R}}$ and mass $\mathbf{M}_{\mathrm{R}}$ matrices are prepared in the next step of the algorithm:

$$\begin{aligned} \mathbf{K}_{\mathrm{R}} &= \mathbf{r}_{\mathrm{B}}^{\mathrm{T}} \mathbf{K}_{\mathrm{M}} \mathbf{r}_{\mathrm{B}}\ . \\ \mathbf{M}_{\mathrm{R}} &= \mathbf{r}_{\mathrm{B}}^{\mathrm{T}} \mathbf{M}_{\mathrm{M}} \mathbf{r}_{\mathrm{B}} \end{aligned} \tag{7}$$

Following this, the first (lowest) eigenvalue λ_i can be computed solving the reduced eigenproblem:

$$\mathbf{K}_{\mathrm{R}}\mathbf{y}_1 = \lambda_1 \mathbf{M}_{\mathrm{R}}\mathbf{y}_1, \tag{8}$$

where: $\mathbf{y}_1$ – vector of coefficients.

Special code of CA algorithm was drawn up and developed for the application in case of performed computations of natural frequencies of structure vibrations in MATLAB Software [9].

Neural networks

Back-propagation neural networks (BPNNs) are proposed for computation of the first natural frequencies of horizontal vibrations of modified medium height load-bearing walls as the second reanalysis approach in this paper.

Neural networks (NNs) belong to a group of biologically inspired computer methods [1, 2, 4]. They are composed of processing units (artificial neurons) and connections. The neural network parameters are associated with characteristics of neurons and weights of connections. The neurons are arranged in layers: input layer, hidden layers, output layer. The procedure of neural network parameters optimization, called learning (or training), is performed using a set of training patterns. The generalization capacity of the trained NN is controlled in the testing phase. Small testing errors confirm the good predictive properties of the network.

Artificial neural networks are widely applied in many fields of sciences and engineering, because of their special features, among others: the ability to learn, generalization of knowledge and parallel processing.

Identification of the modified load-bearing walls natural frequencies

Analysed walls

The influence of the modifications (geometrical changes) on the first natural wall frequency was analysed in thecase of typical medium-height reinforced concrete load-bearing walls with 2.7 m, 5.4 m, 11.7 m width and 14 m (5 storeys x 2.8 m) height, 0.14 m thickness.

The modifications in the form of single door opening and system door openings – a series of door openings one above another on all storeys – were introduced to the walls and considered.

The widths of door openings were taken from the range of 0.9 m – 4.8 m with a 0.3 m step. The door openings' different positions were considered. Door openings were "shifted" from the wall's edge with a 0.3 m step. The examples of analysed walls (with finite element method mesh) are shown in Figure 1.

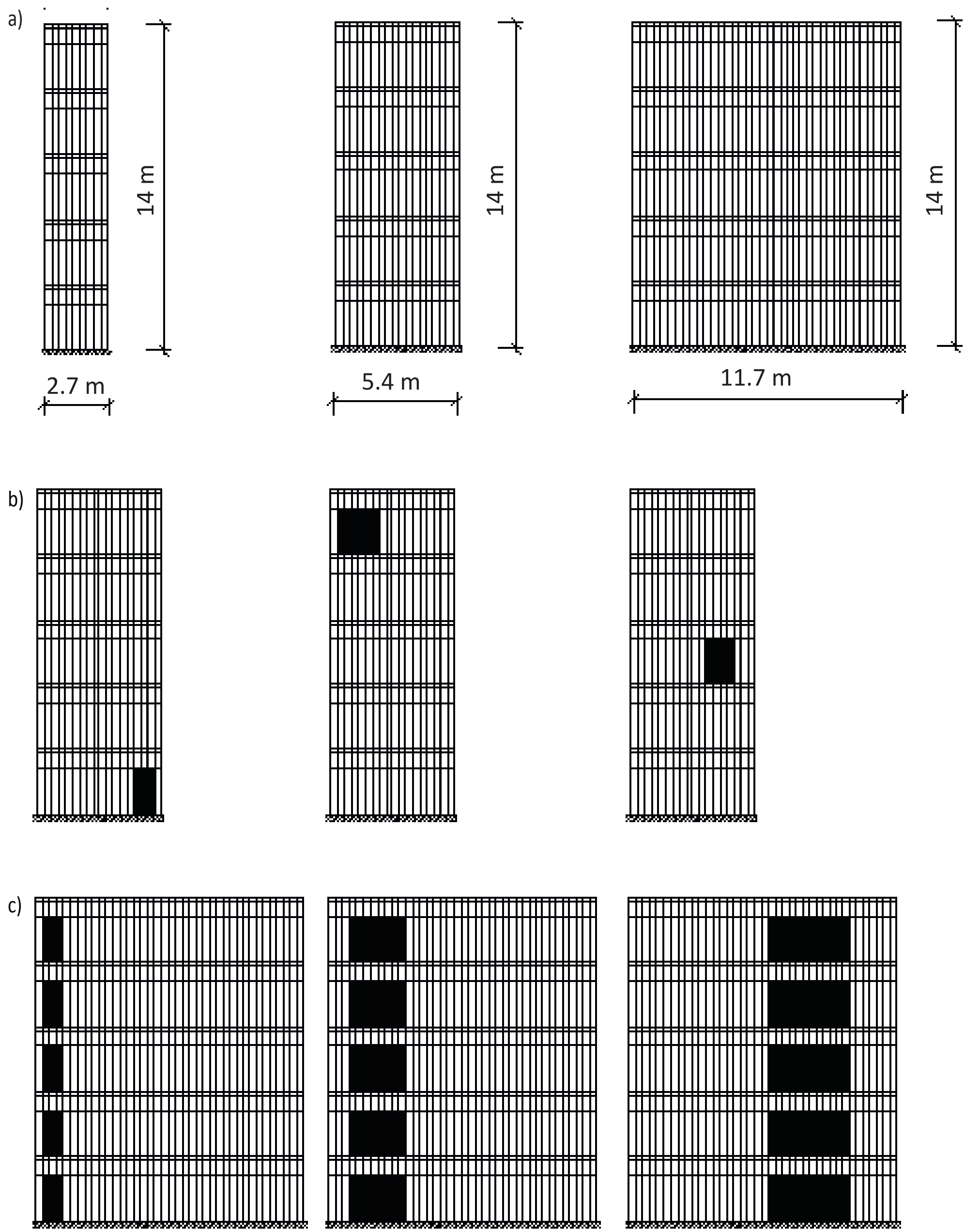

Fig. 1. Analysed walls: a) walls before modification; b) examples of walls with modifications in the form of single door opening; c) examples of walls with modifications in the form of system door openings – a series of door openings one above another on all storeys

Computations using the CA method

The accuracy of combined approximations hybrid method algorithm (CA) as the reanalysis approach was verified in evaluation of the first natural frequency of horizontal vibrations of modified walls. CA procedure was applied in case of modifications in the form of single door opening as well as in case of modifications in the form of system door openings – a series of door openings one above another on all storeys. The first one could be treated as a small modification of the wall, whereas the second one – a large modification.

The influence of the number of basis vectors on the accuracy of obtained results was investigated.

To illustrate the accuracy of the results obtained using the CA method, fractions r_1 and relative errors Errf_1 were computed:

$$r_1 = f_{1\mathrm{CA}} / f_{1\mathrm{FEM}} , \tag{9}$$

$$\mathrm{Err}f_1 = \frac{\left| f_{1\mathrm{FEM}} - f_{1\mathrm{CA}} \right|}{f_{1\mathrm{FEM}}} \cdot 100\% , \tag{10}$$

where: $f_{1\mathrm{CA}}$, $f_{1\mathrm{FEM}}$ – the first natural frequency of wall vibrations computed using CA method and FEM, respectively.

Implemented neural networks

The relation between geometrical changes and structural parameters, and the first natural frequencies of horizontal vibrations of the modified walls (f_1), was determined using back-propagation neural networks (BPNNs). Neural networks were applied as the reanalysis tools. The networks were trained by means of MATLAB NN Toolbox using Levenberg-Marquadt learning algorithm and sigmoid activation function [3]. Finite element method (FEM) – code Ansys [10] was applied to generate neural networks patterns according to considered cases of the modifications and the fact of symmetry in cases of door openings positions was taken into account.

The following parameters were considered as the input information: p_1, p_2 – coordinates of the door opening location, b_1 – door opening width, b_2 – wall width, f_{1S} – the first natural frequency of the wall without door openings. The geometrical parameters considered in the input vectors of the proposed neural networks are shown in Figure 2.

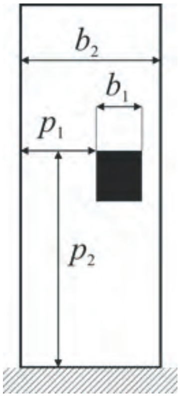

Fig. 2. Parameters considered in the input vectors of the proposed neural networks

Various combinations of network input parameters were discussed. Finally, three variants of neural networks are proposed for reanalysis of the walls with single door opening (BPNNs: NNS1, NNS2, NNS3), and also three variants of networks for reanalysis of the walls with system door openings introduced as the modifications (BPNNs: NNL4, NNL5, NNL6). The input parameters and the structures of the proposed neural networks are shown in Table 1. The prepared patterns were split into three sets: training (60%), validating (about 20%), testing (about 20%) in all cases of neural network structures (architectures).

Tab. 1. Input parameters and architectures of proposed neural networks

BPNN	Input parameters	BPNN architecture
NNS1	p_1, p_2, b_1, b_2	4-27-1
NNS2	$p_1, p_2, b_1, b_2, f_{1S}$	5-27-1
NNS3	p_1, p_2, b_1, f_{1S}	4-27-1
NNL4	p_1, b_1, b_2	3-11-1
NNL5	p_1, b_1, b_2, f_{1S}	4-13-1
NNL6	p_1, b_1, f_{1S}	3-15-1

Numerical results

The reanalysis results obtained using the CA method as well as neural networks and results from solving the full eigenproblem by finite element method (FEM) were compared. Values of the first natural frequencies obtained in case of some examples of modified walls are presented in Table 2 (single door opening) and Table 3 (system door openings). Frequencies shown in Table 2 and Table 3 were computed using only 5 basis vectors in the CA method. It is also worth noting that the presented values of frequencies determined using BPNNs relate to testing patterns of the networks.

Tab. 2. Values of the first natural frequencies obtained in case of some examples of modified walls with single door opening using FEM, CA and BPNNs

Walls					FEM	CA (5 vec.)	BPNNs		
b_2 [m], f_{1S} [Hz]	No.	b_1 [m]	p_1 [m]	p_2 [m]			NNS1	NNS2	NNS3
b_2 = 2.7 m f_{1S} = 4.41 Hz	S1.1	0.9	0.3	4.8	4.08	4.08	4.09	3.99	4.17
	S1.2	0.9	0.6	4.8	4.27	4.27	4.27	4.23	4.30
	S1.3	0.9	0.3	10.4	4.40	4.40	4.38	4.32	4.38
	S1.4	0.9	0.6	10.4	4.42	4.42	4.43	4.44	4.43
	S1.5	0.9	0.9	10.4	4.42	4.42	4.44	4.44	4.43
	S1.6	1.2	0.3	4.8	4.02	4.02	3.99	4.00	4.07
	S1.7	1.2	0.3	10.4	4.39	4.39	4.36	4.36	4.37
	S1.8	1.2	0.6	10.4	4.41	4.41	4.42	4.41	4.41
	S1.9	1.5	0.3	7.6	4.18	4.18	4.19	4.15	4.18
	S1.10	1.5	0.6	7.6	4.25	4.25	4.28	4.22	4.25
b_2 = 5.4 m f_{1S} = 8.28 Hz	S2.1	0.9	0.9	4.8	7.97	7.97	7.97	7.97	7.97
	S2.2	0.9	0.9	10.4	8.26	8.26	8.26	8.27	8.26
	S2.3	1.2	0.9	2.0	7.86	7.86	7.88	7.88	7.87
	S2.4	2.1	1.5	2.0	8.06	8.06	8.04	8.08	8.05
	S2.5	2.4	0.9	4.8	7.47	7.47	7.48	7.44	7.46
	S2.6	1.5	0.3	10.4	8.18	8.18	8.17	8.14	8.19
	S2.7	2.1	1.5	10.4	8.16	8.16	8.15	8.18	8.17
	S2.8	2.4	0.3	10.4	8.03	8.03	8.03	7.94	7.98
	S2.9	1.2	1.2	13.2	8.37	8.37	8.37	8.37	8.37
	S2.10	1.5	0.6	13.2	8.41	8.40	8.42	8.41	8.41
b_2 = 11.7 m f_{1S} = 14.46 Hz	S3.1	0.9	0.3	4.8	14.00	14.00	13.99	14.01	14.06
	S3.2	0.9	1.5	7.6	14.30	14.30	14.30	14.33	14.30
	S3.3	0.9	2.7	10.4	14.40	14.39	14.40	14.38	14.39
	S3.4	3.3	2.1	4.8	13.06	13.07	13.06	13.08	13.05
	S3.5	3.6	2.4	4.8	13.01	13.02	13.01	13.01	13.01
	S3.6	4.2	1.2	4.8	12.13	12.13	12.13	12.11	12.13
	S3.7	4.2	0.3	7.6	12.24	12.25	12.23	12.22	12.31
	S3.8	1.8	3.9	10.4	14.24	14.24	14.24	14.24	14.23
	S3.9	3.6	1.5	10.4	13.90	13.90	13.88	13.89	13.90
	S3.10	3.9	3.3	13.2	14.34	14.35	14.34	14.34	14.35

Tab. 3. Values of the first natural frequencies obtained in case of some examples of modified walls with system door openings using FEM, CA and BPNNs

Walls				FEM	CA (5 vec.)	BPNNs		
b_2 [m], f_{1S} [Hz]	No.	b_1 [m]	p_1 [m]			NNL4	NNL5	NNL6
b_2 = 2.7 m f_{1S} = 4.41 Hz	L1.1	0.9	0.3	3.70	3.70	3.66	3.49	3.69
	L1.2	1.2	0.6	4.08	4.08	4.07	4.10	4.06
	L1.3	1.5	0.3	3.52	3.53	3.55	3.65	3.55
	L1.4	0.9	0.9	4.29	4.29	4.28	4.51	4.29
b_2 = 5.4 m f_{1S} = 8.28 Hz	L2.1	1.2	0.3	6.67	6.68	6.63	6.66	6.67
	L2.2	1.2	1.5	7.38	7.39	7.37	7.36	7.36
	L2.3	1.2	1.8	7.40	7.41	7.39	7.39	7.39
	L2.4	1.2	3.0	7.33	7.34	7.32	7.30	7.32
	L2.5	1.5	0.3	6.29	6.31	6.37	6.34	6.35
	L2.6	1.5	0.6	6.70	6.72	6.71	6.72	6.70
	L2.7	1.5	1.2	6.98	7.00	6.97	6.95	6.94
	L2.8	1.8	0.3	5.91	5.96	6.01	5.98	5.99
	L2.9	2.1	0.3	5.55	5.63	5.63	5.60	5.62
	L2.10	2.1	0.6	5.99	6.05	6.00	5.98	5.95
b_2 = 11.7 m f_{1S} = 14.46 Hz	L3.1	0.9	1.5	13.60	13.60	13.59	13.58	13.59
	L3.2	1.2	3.6	12.72	12.73	12.70	12.70	12.72
	L3.3	1.8	1.2	12.39	12.43	12.37	12.38	12.36
	L3.4	2.1	2.7	11.11	11.18	11.11	11.11	11.10
	L3.5	2.4	2.1	10.87	10.96	10.86	10.86	10.86
	L3.6	2.7	2.7	9.91	10.04	9.92	9.91	9.92
	L3.7	3.3	3.3	8.49	8.71	8.48	8.48	8.49
	L3.8	4.2	1.8	7.99	8.34	7.99	8.00	8.00
	L3.9	4.5	1.5	7.80	8.20	7.79	7.79	7.81
	L3.10	4.8	2.1	6.82	7.33	6.81	6.81	6.83

Examples of fractions $r_1 = f_{1CA} / f_{1FEM}$ depending on the number of basis vectors for the first natural frequency of two of the b_2 = 5.4 m walls and one of the b_2 = 11.7 m walls are shown in Fig. 3 and Table 4, respectively.

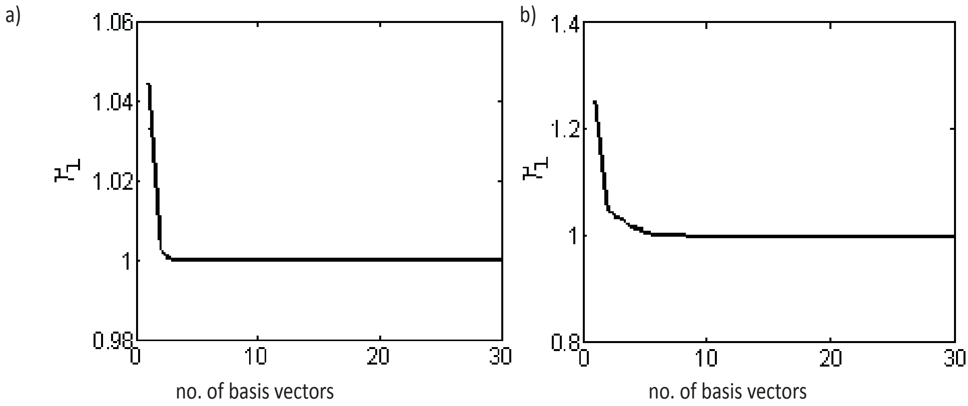

Fig. 3. Fractions $r_1 = f_{1CA} / f_{1FEM}$ depending on the number of basis vectors for the first natural frequency of the walls b_2 = 5.4 m: a) single door opening, b) system doors opening

Tab. 4. Fractions $r_1 = f_{1CA} / f_{1FEM}$ and relative errors Errf_1 depending on the number of basis vectors for the first natural frequency of the wall b_2 = 11.7 m (single door opening)

Number of basis vectors	r_1	**Err f_1 [%]**
1	1.0203	2.0336
2	1.0015	0.1461
3	1.0001	0.0140
4	1.0001	0.0052
5	1.0000	0.0035
10	1.0000	0.0032
15	1.0000	0.0032
20	1.0000	0.0032

It can be seen that CA method gives very good results for both small and large changes in the reanalysed walls. It is worth to mention the good convergence of the computations of the first natural frequency of the modified walls. In all considered cases using only two basis vectors leads to relative error of less than 3%, whereas using five basis vectors makes the relative error less than 0.01%.

In turn, the average relative errors of computations using above mentioned BPNNs are not greater than 0.15% in case of training, 0.25% in case of validating and 0.26% in case of testing processes.

Conclusion

The numerical results show that the CA method is an efficient reanalysis procedure for computing of the first natural vibration frequencies of the modified load-bearing walls with both small and large changes. Decreasing of the number of algebraic operations leads to a reduction of the computational effort, making the analysis much faster with no significant decrease of the accuracy. Also the application of proposed BPNNs enables us to identify the frequencies with very good accuracy – comparable to the CA method.

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Abstract

Two reanalysis techniques for computation of natural frequencies of the modified typical medium-height load-bearing walls are discussed and compared in the paper: combined approximations hybrid method (CA) and back-propagation neural networks (BPNNs). The small and the large changes of the wall stiffness and mass resulting from the new door openings' size and position were analysed. It was stated that both proposed methods enable to identify the first natural frequencies of modified load-bearing walls with very good accuracy.

Key words: neural networks, combined approximations method, reanalysis, modified structure, natural frequency of vibration

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