FOLIA 180

Annales Universitatis Paedagogicae Cracoviensis

Studia Technica VIII (2015)

Miluše Vítečková, Antonín Víteček, Kazimierz Jaracz

Control systems for plants with time delay

Introduction

There are many problems and difficulties with control of plants (technological processes) with time delay. The time delay can appear as a pure transportation time delay or as an approximation of high order inertia. There are different approaches to control of plants with time delay. One of the ways is the usage of a more complex structure of a control system, which is simply realizable and not financially demanding thanks to the development of efficient digital devices. The goal of this paper is to point to some general quality relations among the more important structures of control systems, which are used for plants with dominant time delay.

Classic Control System

Consider the classic closed-loop control system in Fig. 1, where G_C is the conventional controller transfer function, G_PG_D – the plant transfer function, W, U, V and Y – the transforms of the desired, manipulated, disturbance and controlled variables.

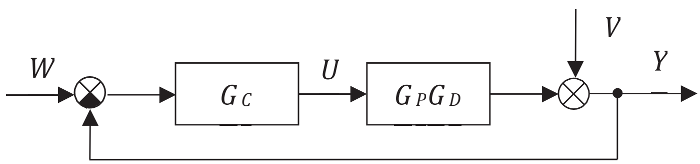

Fig. 1. Closed-loop structure of classic control system

It is considered that the plant transfer function G_PG_D consists of two parts: a proportional minimum phase part G_P (all poles and zeros are stable) and a part G_D, which represents the time delay.

The aim of the synthesis of the classical closed-loop control system in Fig. 1 is the design of a suitable conventional controller with the transfer function G_C, and to tune it so that for the control system the transfer function G_{wy} and the disturbance transfer function G_{vy} hold

$$G_{wy} = \frac{Y}{W} \to 1\,, \tag{1}$$

$$G_{vy} = \frac{Y}{V} = 1 - G_{wy} \to 0\,. \tag{2}$$

The conditions (1) and (2) are too idealized. It is necessary to appreciate that the transfer functions are generally complex functions of complex variables and therefore during the existence of time delay these conditions, for the reason of the causality principle, will have forms [Šulc, Vítečková 2004]

$$G_{wy} \to G_D\,, \tag{3}$$

$$G_{vy} \to 1 - G_D\,. \tag{4}$$

It is obvious that the other conditions connected with the stability and quality of the control process must hold.

Fulfilment of the conditions (1) and (2) or (3) and (4) (it can be partial) is demanded for the operational bandwidth.

Relations (2) and (4) imply that if the disturbance variable influences the plant output, the guarantee of the suitable behaviour of the transfer function G_{wy} [see (1) and (3)] causes the guarantee of the suitable behaviour of the disturbance transfer function G_{vy}. Therefore it will further be dealt only with the control system transfer function G_{wy}.

The scheme in Fig. 1 implies that for the operational bandwidth, for which it is valid that

$$|G_C| \to \infty\,, \tag{5}$$

then the relation

$$G_{wy} = \frac{G_P G_D}{\dfrac{1}{G_C} + G_P G_D} \to 1 \tag{6}$$

will be simultaneously true.

Condition (5) can be ensured for a conventional controller by a suitable combination of proportional, integral and derivative components. From relation (6) it is obvious that if condition (5) holds, then the closed-loop structure of the classical control system in Figure 1 realizes an inverse of the feedback (unit in this case).

No problems with time delay are directly seen from the closed-loop structure of a classic control system [compare relations (6) and (3)]. Therefore it is useful to transform the structure in Figure 1 into the equivalent, open-loop structure of the classic control system, as shown in Figure 2.

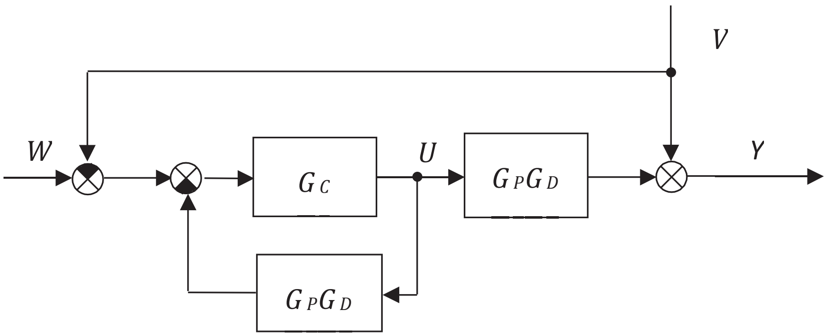

Fig. 2. Equivalent open-loop structure of classic control system in Fig. 1

On the basis of Figure 2, for the open-loop structure of the classic control system it can be written

$$G_{wy} = G_{wu} G_P G_D , \tag{7}$$

$$G_{wu} = \frac{1}{\frac{1}{G_C} + G_P G_D} , \tag{8}$$

where

$$G_{wu} = \frac{U}{W} \tag{9}$$

is the transfer function for the manipulated variable.

If the plant does not contain time delay, i.e.

$$G_D = 1 , \tag{10}$$

then for the operational bandwidth for which (5) is valid, fulfilment of the conditions (1) and (2) is relatively easy. On the basis of (8) for (5) and (10) it can be obtained

$$G_{wu} \to \frac{1}{G_P} , \tag{11}$$

and therefore in accordance with (7) the relations (1) and (2) hold.

From relation (11) it is obvious that the inverse of feedback is realized, i.e. the inverse of the transfer function G_P, which represents the behaviour of the real plant. Because the plant is a minimum phase process, the modulus of the conventional controller transfer function $|G_C|$ can have a very high value and therefore the inversion (11) in the operational bandwidth can be realized relatively precisely. For this reason the classic closed-loop control system for the proportional minimum phase plant without time delay for a correctly tuned conventional controller can guarantee a sufficient quality and robustness of the control process.

If the plant contains the time delay, i.e.

$$G_D \neq 1\,, \tag{12}$$

then the relation (8) implies that for the reason of noninvertibility of time delay, it is not possible to hold condition (5). Because the inverse of the behaviour of the plant with time delay is not possible to realize in the operational bandwidth, therefore the quality of the control process in the case of the existence of the dominant time delay and usage of conventional controllers will be low. In addition to this, special approaches and methods must be used for the synthesis [Górecki 1971; Górecki et al. 1989; Šulc, Vítečková 2004; Wade 2004; Normey-Rico, Camacho 2007].

Control System with Smith Predictor

The structure in Figure 2 has a basic disadvantage, which is the existence of noninvertible time delay in the feedback surrounding the conventional controller. One of the ways of removing the time delay from this feedback is the use of a more complex structure of the control system (Fig. 3), which leads to the control system with Smith predictor with the transfer function [Górecki 1971; Górecki et al. 1989; Wade 2004; Normey-Rico, Camacho 2007]:

$$G_C^{SP} = \frac{G_C}{1 + G_C G_{PM}\left(1 - G_{DM}\right)}\,, \tag{13}$$

where G_{PM} is the model transfer function of the part G_P, G_{DM} – the model transfer function of the time delay.

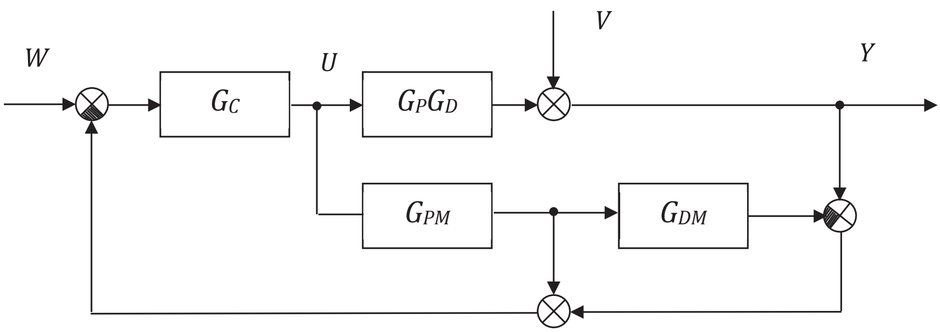

Fig. 3. Structure of control system with Smith predictor

From Figure 3 it is obvious that one part of additional branch ensures the removal of time delay from the feedback and the second part of the branch identifies the disturbance. This scheme can be substituted for the closed-loop structure of the classical control system in Figure 1 or by an equivalent scheme of the open-loop structure of the classical control system in Figure 2, where the conventional controller with the transfer function G_C will be replaced by the Smith predictor with the transfer function (13).

The transfer function for the manipulated variable in the control system with the Smith predictor is given by the relation

$$G_{wu} = \frac{1}{\dfrac{1}{G_C} + G_{PM} + G_P G_D - G_{PM} G_{DM}} . \tag{14}$$

If the model of the plant with time delay will be identical to the real plant, i.e.

$$G_{PM} = G_P , \qquad G_{DM} = G_D , \tag{15}$$

then the control system with the Smith predictor (Fig. 3) can be presented by the closed-loop structure (Fig. 4) or the open-loop structure (Fig. 5) of the classical control system with a conventional controller.

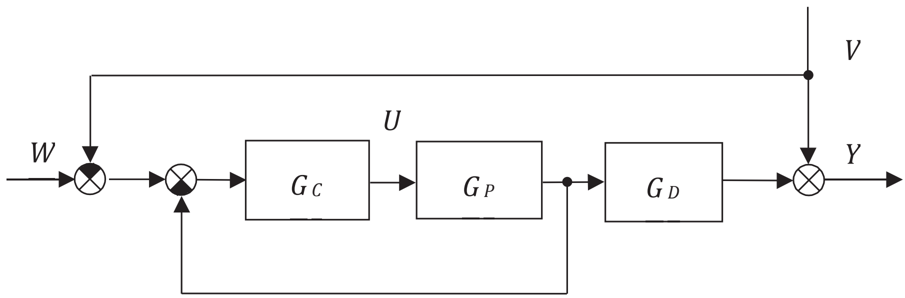

Fig. 4. Closed-loop control system with Smith predictor if condition (15) holds

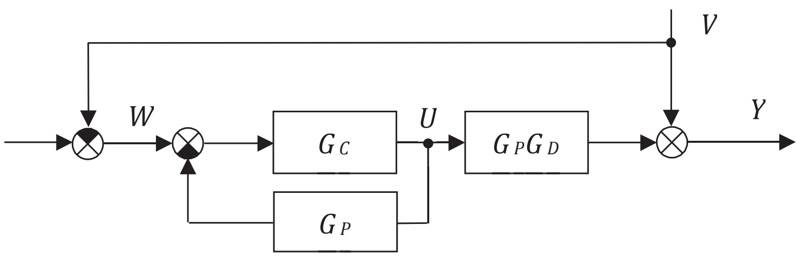

Fig. 5. Open-loop control system with Smith predictor if condition (15) holds

If condition (15) from Figure 4 holds, it implies that for tuning of the control system with the Smith predictor it is possible to use all tuning methods for classical control systems without time delay.

From Figures 4 and 5 it is obvious that the time delay was really removed from the feedback. If conditions (15) and (5) hold, then the relation (11) will be true and therefore in accordance with relation (7), the relations (3) and (4) will hold too. These conclusions hold evidently only for an ideal equality between the model and the real plant, see (15).

In real conditions an ideal equality is impossible and therefore, if condition (5) holds, from relations (14) and (7) it can be obtained

$$G_{wu} \to \frac{1}{G_{PM} + G_P G_D - G_{PM} G_{DM}}, \tag{16}$$

$$G_{wy} \to \frac{G_P}{G_{PM} + G_P G_D - G_{PM} G_{DM}} G_D. \tag{17}$$

The requirement (5) of the high value of the modulus of the conventional controller transfer function $|G_C|$ can be fulfilled if the difference between the model and the real plant is not major, because the terms comprising the time delay in the denominator for the transfer function for the manipulated variable (14) are additive to the model of the proportional part of the plant without time delay G_{PM}, see (17) too.

The basic difference between the classical control system without time delay [Fig. 1 and 2 for (10)] and the control systems with the Smith predictor [Fig. 4 and 5 or Fig. 3 for (15)] is not only for the reason of the causality principle in the control system with the Smith predictor, the response must be delayed for the time delay of the plant, but that in the classic control system without time delay the inverse of the behaviour of the plant is realized on the basis of the real plant, i.e. G_P. On the other hand, in the control system with a Smith predictor this inversion is realized on the basis of its model, i.e. G_{PM}. Therefore a robustness of the control system with a Smith predictor will be lower than a robustness of the classic control system without time delay [Martins de Carvalho 1993].

Internal Model Control

In the control system with the Smith predictor the inverse of the behaviour of the proportional part of the plant was realized on the basis of its model G_{PM} by means of negative feedback. This inversion can be realized directly by the usage of the internal model control (IMC) structure in Figure 6, which leads to the internal model controller with a transfer function [Rivera et al. 1986, Wade 2004]

$$G_C^{IM} = \frac{G_F}{G_{PM}(1 - G_F G_{DM})}, \tag{18}$$

where G_F is the transfer function of a suitable chosen low-pass filter.

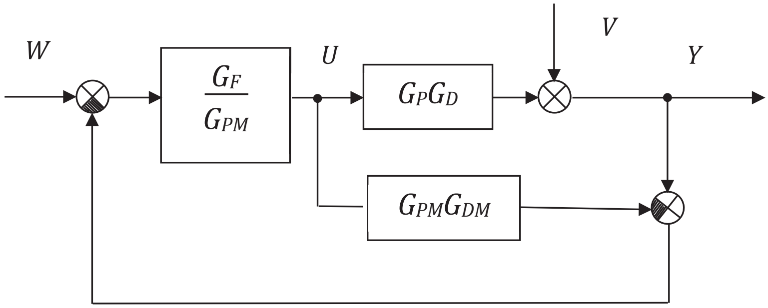

Fig. 6. Internal model control structure

From Figure 6 it is obvious that the branch in the control system with an internal model serves the identification of the disturbance. The filter with the transfer function G_F ensures the causality of the internal model controller.

The structure of the internal model control in Figure 6 can be a substitute for the scheme of the closed-loop structure (Fig. 1) or the scheme of the equivalent open-loop structure (Fig. 2) of the classical control system, where the conventional controller with the transfer function G_C will be replaced by the internal model controller with the transfer function (18).

In the case of an ideal equality of the model with the real controlled process, i.e., if condition (15) holds, then the structure in Figure 6 can be presented as the open-loop structure of the control system with the internal model controller, as shown in Fig. 7. Fig. 7 implies that the transfer function of the manipulated variable is given by a formula

$$G_{wu} = \frac{G_F}{G_P} \tag{19}$$

and in accordance with relation (7), the transfer function of the control system is

$$G_{wy} = G_F G_D\,. \tag{20}$$

It is obvious that the basic difference between the Smith predictor control and the internal model control is that the Smith predictor realizes the inverse of the proportional part of the plant by means of the negative feedback (see Fig. 5), on the other hand the internal model controller directly uses the inverse of the proportional part of the plant and simultaneously, by the means of a suitable selected filter, ensures its causality (Fig. 7).

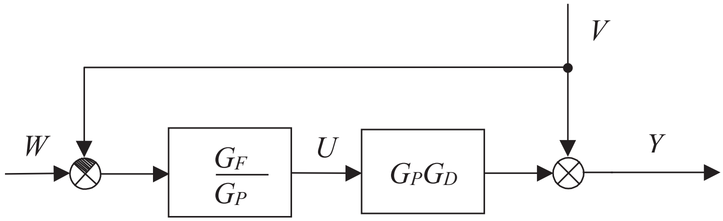

Fig. 7. Open-loop structure of internal model control if condition (15) holds

In comparison of the transfer function of the Smith predictor (13) with the transfer function of the internal model controller (18), if condition (5) holds, the relation

$$G_C^{SP} \to G_C^{IM} \qquad \text{for} \qquad G_F = 1 \tag{21}$$

will be true.

It is obvious that in real conditions the quality of control for a correctly chosen filter in the case of the use of the internal model controller should be higher.

If the conditions of equality of the model with the real controlled process (15) will not be true, then the relations (19) and (20) will not be true too and relations

$$G_{wu} = \frac{G_F}{G_{PM} + G_F\left(G_P G_D - G_{PM} G_{Dm}\right)}, \tag{22}$$

$$G_{wy} = \frac{G_F G_P}{G_{PM} + G_F\left(G_P G_D - G_{PM} G_{DM}\right)} G_D \tag{23}$$

will hold.

From both latest relations it is implied that by a suitable selection of a filter, it is possible to suppress influence of the terms containing time delay mainly for higher frequencies. Thus the filter has two functions: to guarantee the causality of the internal model controller and to damp the adverse influence of the inaccuracy of a model of the controlled process.

Conclusion

The paper points to some more important, mainly qualitative properties of the basic approaches to controlling the processes with dominant time delay on the basis of a rather unusual equivalent open-loop structure of the control system. Most of the conclusions can be extended to very complex controlled processes, where the transfer function G_P represents their invertible part and the transfer function G_D their stable noninvertible part.

It is possible to find the other details and potential approaches to the control of the processes with time delay in the publications [Górecki et al. 1989; Martins de Carvalho 1993; Wade 2004; Normey-Rico – Camacho 2007; Zítek – Víteček 1999].

The research work was supported by research project GACR No P101/12/2520.

References

[1] Górecki H., *Analiza i synteza układów regulacji z opóźnieniem*. Wydawnictwo Naukowo--Techniczne, Warszawa 1971.

[2] Górecki H., Fuksa S., Grabowski P., Korytowski A., *Analysis and Synthesis of Time Delay Systems. Polish Scientific Publishers*, Warszawa 1989.

[3] Martins de Carvalho J.L., *Dynamical Systems and Automatic Control*. Prentice Hall International, London 1993.

[4] Normey-Rico J.E., Camacho E.F., *Control of Dead-time Processes*. Springer-Verlag, London 2007.

[5] Rivera D.E., Morari M., Skogestad S., Internal Model Control. 4. PID Controller Design. Ind. Eng. Chem. Process Des. Dev. 1986, 25(1), p. 252–265.

[6] Šulc B., Vítečková M., *Teorie a praxe návrhu regulačních obvodů*. Vydavatelství ČVUT, Praha 2004.

[7] Wade H.L., *Basic and Advanced Regulatory Control: System Design and Applications*. Second Edition. ISA – The Instrument, Systems, and Automation Society, Research Triangle Park, North Carolina 2004.

[8] Zítek P., Víteček A., *Návrh řízení podsystémů se zpožděními a nelinearitami*. Vydavatelství ČVUT v Praze, Praha 1999.

Abstract

The paper deals with a control of plants with time delay. Two of the most important control systems, which are used for control of plants with dominant time delay, are compared and their qualitative properties are shown. The goal of the paper is to help control system designers with control synthesis for plants with time delay.

Key words: time delay, Smith predictor, internal model control (IMC)

Miluše Vítečková, Antonín Víteček
VSB – Uniwersytet Techniczny Ostrawa
Republika Czeska

Kazimierz Jaracz
Pedagogical University of Cracow
Institute of Technology
ul. Podchorążych 2
30-084 Kraków, Poland