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Nanjundan Magesh and Jagadeesan Yamini

Fekete-Szegö inequalities associated with k^{th} root transformation based on quasi-subordination

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Abstract. Recently, Haji Mohd and Darus [1] revived the study of coefficient problems for univalent functions associated with quasi-subordination. Inspired largely by this article, we provide coefficient estimates with k-th root transform for certain subclasses of $\mathcal{S}$ defined by quasi-subordination.

1. Introduction

Denote by $\mathcal{A}$ the class of all analytic functions of the type

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (z \in \mathbb{U}), \tag{1}$$

where $\mathbb{U} = \{z \in \mathbb{C} : \ |z| < 1\}$. Also denote by $\mathcal{S}$ the class of all analytic univalent functions of the form (1) in $\mathbb{U}$. Let k be a positive integer. A domain $\mathbb{D}$ is said to be k-fold symmetric if a rotation of $\mathbb{D}$ about the origin through an angle $\frac{2\pi}{k}$ carries $\mathbb{D}$ to itself. A function f is said to be k-fold symmetric in $\mathbb{U}$, if $f(e^{\frac{2\pi i}{k}} z) = e^{\frac{2\pi i}{k}} f(z)$ for every $z \in \mathbb{U}$. If f is regular and k-fold symmetric in $\mathbb{U}$, then

$$f(z) = b_1 z + b_{k+1} z^{k+1} + b_{2k+1} z^{2k+1} + \ldots. \tag{2}$$

Conversely, if f is given by (2), then f is k-fold symmetric inside the circle of convergence of the series. For $f \in \mathcal{S}$ given by (1), the k^{th} root transformation is defined by

$$F(z) = [f(z^k)]^{\frac{1}{k}} = z + b_{k+1} z^{k+1} + b_{2k+1} z^{2k+1} + \ldots. \tag{3}$$

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Corresponding author N. Magesh (nmagi_2000@yahoo.co.in).

For two analytic functions f and g, the function f is quasi-subordinate to g in the open unit disc $\mathbb{U}$, if there exist analytic functions h and w, with $|h(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$, such that $\frac{f(z)}{h(z)}$ is analytic in $\mathbb{U}$ and written as

$$\frac{f(z)}{h(z)} \prec g(z) \qquad (z \in \mathbb{U})$$

and it is denoted by

$$f(z) \prec_q g(z) \qquad (z \in \mathbb{U})$$

and equivalently

$$f(z) = h(z)g(w(z)) \qquad (z \in \mathbb{U}).$$

It is interesting to note that if $h(z) \equiv 1$, then $f(z) = g(w(z))$, so that $f(z) \prec g(z)$ in $\mathbb{U}$, where $\prec$ is a subordination between f and g in $\mathbb{U}$. Also notice that if $w(z) = z$, then $f(z) = h(z)g(z)$ and it is said that f is majorized by g and written as $f(z) \ll g(z)$ in $\mathbb{U}$ (see [2]).

Let φ be an analytic and univalent function with positive real part in $\mathbb{U}$, $\varphi(0) = 1$, $\varphi'(0) > 0$ and let φ map the unit disk $\mathbb{U}$ onto a region starlike with respect to 1 and symmetric with respect to the real axis. The Taylor's series expansion of such a function is

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \dots, \tag{4}$$

where all coefficients are real and $B_1 > 0$.

Recently, El-Ashwah and Kanas [3] introduced and studied the following two subclasses:

$$\mathcal{S}_q^*(\gamma, \varphi) := \Big\{ f \in \mathcal{A} : \ \frac{1}{\gamma}\Big(\frac{zf'(z)}{f(z)} - 1\Big) \prec_q \varphi(z) - 1, \ z \in \mathbb{U}, \ \gamma \in \mathbb{C} \setminus \{0\} \Big\}$$

and

$$\mathcal{K}_q(\gamma, \varphi) := \Big\{ f \in \mathcal{A} : \ \frac{1}{\gamma}\frac{zf''(z)}{f'(z)} \prec_q \varphi(z) - 1, \ z \in \mathbb{U}, \ \gamma \in \mathbb{C} \setminus \{0\} \Big\}.$$

We note that, when $h(z) \equiv 1$, the classes $\mathcal{S}_q^*(\gamma, \varphi)$ and $\mathcal{K}_q(\gamma, \varphi)$ reduce respectively, to the familiar classes $\mathcal{S}^*(\gamma, \varphi)$ and $\mathcal{K}(\gamma, \varphi)$ of Ma-Minda starlike and convex functions of complex order γ ($\gamma \in \mathbb{C}\setminus\{0\}$) in $\mathbb{U}$ (see [4]). For $\gamma = 1$, the classes $\mathcal{S}_q^*(\gamma, \varphi)$ and $\mathcal{K}_q(\gamma, \varphi)$ reduce to the classes $\mathcal{S}_q^*(\varphi)$ and $\mathcal{K}_q(\varphi)$ studied by Haji Mohd and Darus [1]. When $h(z) \equiv 1$, the classes $\mathcal{S}_q^*(\varphi)$ and $\mathcal{K}_q(\varphi)$ reduce respectively, to well known subclasses $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ introduced and studied by Ma and Minda [5]. By specializing

$$\varphi(z) = \frac{1 + (1 - 2\alpha)z}{1 - z} \qquad (0 \leq \alpha < 1)$$

or

$$\varphi(z) = \Big(\frac{1 + z}{1 - z}\Big)^{\beta} \qquad (0 < \beta \leq 1)$$

the classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ consist of functions known as the starlike (respectively convex) functions of order α or strongly starlike (respectively convex) functions of order β, respectively.

A function $f \in \mathcal{A}$ given by (1) is said to be in the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$, $0 \neq \gamma \in \mathbb{C}$, $\delta \geq 0$, if the following quasi-subordination condition is satisfied

$$\frac{1}{\gamma}\Big((1-\delta)\frac{z\mathcal{F}'_\lambda(z)}{\mathcal{F}_\lambda(z)} + \delta\Big(1+\frac{z\mathcal{F}''_\lambda(z)}{\mathcal{F}'_\lambda(z)}\Big) - 1\Big) \prec_q \varphi(z) - 1 \qquad (z \in \mathbb{U}),$$

where

$$\mathcal{F}_\lambda(z) = (1-\lambda)f(z) + \lambda z f'(z) \qquad (0 \leq \lambda \leq 1).$$

We note that,

1. $\mathcal{M}_q^{\delta,0}(\gamma,\varphi) := \mathcal{M}_q^{\delta}(\gamma,\varphi)$,
2. $\mathcal{M}_q^{\delta}(1,\varphi) := \mathcal{M}_q^{\delta}(\varphi)$, [1, Definition 1.7, p.3],
3. $\mathcal{M}_q^{0,0}(\gamma,\varphi) := \mathcal{S}_q^{*}(\gamma,\varphi)$, [3, Definition 1.1, p.680],
4. $\mathcal{S}_q^{*}(1,\varphi) := \mathcal{S}_q^{*}(\varphi)$, [1, Definition 1.1, p.2],
5. $\mathcal{M}_q^{1,0}(\gamma,\varphi) := \mathcal{K}_q(\gamma,\varphi)$, [3, Definition 1.3, p.681],
6. $\mathcal{K}_q(1,\varphi) := \mathcal{K}_q(\varphi)$, [1, Definition 1.3, p.2],
7. For $0 \neq \gamma \in \mathbb{C}$, $0 \leq \lambda \leq 1$,

$$\begin{aligned}\mathcal{M}_q^{0,\lambda}(\gamma,\varphi) &\equiv \mathcal{P}_q(\gamma,\lambda,\varphi)\\ &= \Big\{f \in \mathcal{A} : \frac{1}{\gamma}\Big(\frac{zf'(z)+\lambda z^2 f''(z)}{(1-\lambda)f(z)+\lambda z f'(z)} - 1\Big) \prec_q \varphi(z) - 1,\ z \in \mathbb{U}\Big\}.\end{aligned}$$

8. For $0 \neq \gamma \in \mathbb{C}$, $0 \leq \lambda \leq 1$,

$$\begin{aligned}&\mathcal{M}_q^{1,\lambda}(\gamma,\varphi)\\ &\quad\equiv \mathcal{K}_q(\gamma,\lambda,\varphi)\\ &\quad= \Big\{f \in \mathcal{A} :\ \frac{1}{\gamma}\Big(\frac{zf'(z)+(1+2\lambda)z^2 f''(z)+\lambda z^3 f'''(z)}{zf'(z)+\lambda z^2 f''(z)} - 1\Big) \prec_q \varphi(z) - 1,\\ &\qquad\qquad z \in \mathbb{U}\Big\}.\end{aligned}$$

Inspired by the papers of [1, 3, 6, 7, 8], we obtain the upper bounds $|b_{k+1}|$ and $|b_{2k+1}|$ for $f \in \mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$. Also, we investigate the Fekete-Szegö results for the class $\mathcal{M}_q^{\delta,\lambda}(\gamma,\varphi)$ and its special cases. In order to discuss our results we provide the following lemmas.

Lemma 1.1 ([9])
Let w be an analytic function with $w(0) = 0$, $|w(z)| < 1$ and let

$$w(z) = u_1 z + u_2 z^2 + \dots \qquad (z \in \mathbb{U}). \tag{5}$$

Then for $t \in \mathbb{C}$,

$$|u_2 - t u_1^2| \leq \max[1; |t|].$$

Lemma 1.2 ([9])
Let h be an analytic function with $|h(z)| < 1$ and let

$$h(z) = h_0 + h_1 z + h_2 z^2 + \dots \qquad (z \in \mathbb{U}). \tag{6}$$

Then

$$|h_0| \le 1 \quad \text{and} \quad |h_n| \le 1 - |h_0|^2 \le 1 \qquad (n > 0).$$

Lemma 1.3 ([10])
Let w be the analytic function with $w(0) = 0$, $|w(z)| < 1$ and given by (5). Then $|w_1| \le 1$ and for any integer $n \ge 2$,

$$|u_n| \le 1 - |u_1|^2.$$

2. Main result

Unless otherwise stated, throughout the sequel, we set f is of the form (1) and φ, h and w are given by (4), (6) and (5), respectively.

In the following theorem, we find Fekete-Szgeö result for $f \in \mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$.

Theorem 2.1
Let $f \in \mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$ and let F be given by (3). Then

$$|b_{k+1}| \le \frac{|\gamma| B_1}{k(1+\delta)(1+\lambda)},$$

$$|b_{2k+1}| \le \frac{|\gamma|\{B_1 + \max\{B_1, |\frac{\gamma(1+3\delta)k + (1-k)\gamma(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2}|B_1^2 + |B_2|\}\}}{2k(1+2\delta)(1+2\lambda)}$$

and for $\mu \in \mathbb{C}$,

$$|b_{2k+1} - \mu b_{k+1}^2| \le \frac{|\gamma|\{B_1 + \max\{B_1, |\frac{\gamma(1+3\delta)k + (1-2\mu-k)\gamma(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2}|B_1^2 + |B_2|\}\}}{2k(1+2\delta)(1+2\lambda)}.$$

Proof. Since $f \in \mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$, there exist φ and w with

$$|\varphi(z)| \le 1, \quad w(0) = 0 \quad \text{and} \quad |w(z)| < 1$$

such that

$$\frac{1}{\gamma}\Big((1-\delta)\frac{z\mathcal{F}_\lambda'(z)}{\mathcal{F}_\lambda(z)} + \delta\Big(1 + \frac{z\mathcal{F}_\lambda''(z)}{\mathcal{F}_\lambda'(z)}\Big) - 1\Big) = h(z)(\varphi(w(z)) - 1) \tag{7}$$

and

$$h(z)(\varphi(w(z)) - 1) = h_0 B_1 u_1 z + [h_1 B_1 u_1 + h_0(B_1 u_2 + B_2 u_1^2)]z^2 + \dots. \tag{8}$$

From (7) and (8) we get

$$\frac{1}{\gamma}(1+\delta)(1+\lambda)a_2 = h_0 B_1 u_1 \tag{9}$$

and

$$\frac{1}{\gamma}\left[2(1+2\delta)(1+2\lambda)a_3-(1+3\delta)(1+\lambda)^2a_2^2\right]=h_1B_1u_1+h_0B_1u_2+h_0B_2u_1^2. \quad (10)$$

Equation (9) yields

$$a_2=\frac{\gamma h_0B_1u_1}{(1+\delta)(1+\lambda)}. \quad (11)$$

By subtracting (10) from (9) and using (11) we obtain

$$a_3=\frac{\gamma}{2(1+2\delta)(1+2\lambda)}\left[h_1B_1u_1+h_0B_1u_2+\left(h_0B_2+\frac{\gamma h_0^2B_1^2(1+3\delta)}{(1+\delta)^2}\right)u_1^2\right]. \quad (12)$$

For a given $f\in\mathcal{S}$ of the form (1), we define F by

$$\begin{aligned}F(z)&=[f(z^k)]^{\frac{1}{k}}\\&=z+\frac{a_2}{k}z^{k+1}+\left[\frac{a_3}{k}-\left(\frac{k-1}{2k^2}\right)a_2^2\right]z^{2k+1}+\dots\\&=z+b_{k+1}z^{k+1}+b_{2k+1}z^{2k+1}+\dots,\end{aligned}$$

where

$$b_{k+1}=\frac{a_2}{k},\quad b_{2k+1}=\frac{a_3}{k}-\left(\frac{k-1}{2k^2}\right)a_2^2\quad\text{and so on.} \quad (13)$$

It follows from (11), (12) and (13) that

$$b_{k+1}=\frac{a_2}{k}=\frac{\gamma h_0B_1u_1}{k(1+\delta)(1+\lambda)}$$

and

$$\begin{aligned}b_{2k+1}&=\frac{a_3}{k}-\left(\frac{k-1}{2k^2}\right)a_2^2\\&=\frac{\gamma[h_1B_1u_1+h_0B_1u_2+(h_0B_2+\frac{\gamma h_0^2B_1^2(1+3\delta)}{(1+\delta)^2})u_1^2]}{2k(1+2\delta)(1+2\lambda)}\\&\quad-\left(\frac{k-1}{2k^2}\right)\frac{\gamma^2h_0^2B_1^2u_1^2}{(1+\delta)^2(1+\lambda)^2}.\end{aligned}$$

For $\mu\in\mathbb{C}$ we get

$$\begin{aligned}&b_{2k+1}-\mu b_{k+1}^2\\&\quad=\frac{\gamma B_1}{2k(1+2\delta)(1+2\lambda)}\Bigg\{h_1u_1+h_0\Bigg(u_2+\Bigg[\frac{B_2}{B_1}+\frac{\gamma h_0B_1(1+3\delta)}{(1+\delta)^2}\\&\qquad-\frac{\gamma h_0B_1(1+2\delta)(1+2\lambda)}{(1+\delta)^2(1+\lambda)^2}+\frac{\gamma h_0B_1(1-2\mu)(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2}\Bigg]u_1^2\Bigg)\Bigg\}.\end{aligned}$$

Since h is analytic and bounded in $\mathbb{U}$ we have

$$|h_n|\le 1-|h_0|^2\le 1\qquad(n>0).$$

By using this fact and the well-known inequality

$$|u_1| \leq 1,$$

from Lemma 1.3, we conclude that

$$|b_{k+1}| \leq \frac{|\gamma| B_1}{k(1+\delta)(1+\lambda)}$$

and

$$\begin{aligned}&|b_{2k+1} - \mu b_{k+1}^2| \\ &\quad\leq \frac{|\gamma| B_1}{2k(1+2\delta)(1+2\lambda)} \Big\{ 1 + \Big| u_2 - \Big[\frac{-B_2}{B_1} \\ &\quad - \frac{\gamma(1+3\delta)k - \gamma(1+2\delta)(1+2\lambda)k + \gamma(1-2\mu)(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2} h_0 B_1 \Big] u_1^2 \Big| \Big\}.\end{aligned}$$

In view of Lemma 1.1, we have

$$|b_{2k+1} - \mu b_{k+1}^2| \leq \frac{|\gamma| \{ B_1 + \max\{ B_1, |\frac{\gamma(1+3\delta)k + (1-2\mu-k)\gamma(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2}| B_1^2 + |B_2| \} \}}{2k(1+2\delta)(1+2\lambda)}.$$

When $\mu = 0$, we obtain

$$|b_{2k+1}| \leq \frac{|\gamma| \{ B_1 + \max\{ B_1, |\frac{\gamma(1+3\delta)k + (1-k)\gamma(1+2\delta)(1+2\lambda)}{k(1+\delta)^2(1+\lambda)^2}| B_1^2 + |B_2| \} \}}{2k(1+2\delta)(1+2\lambda)}.$$

Hence we obtained the required inequalities of Theorem 2.1.

3. Concluding remarks and corollaries

In light of the special subclasses of the class $\mathcal{M}_q^{\delta,\lambda}(\gamma, \varphi)$, we have the following corollaries and remarks.

Remark 3.1
For $\delta = \lambda = 0$ and $\gamma = 1$, Theorem 2.1 reduces to [6, Theorem 2.1, p.619]. For $\delta = \lambda = 0$ and $\gamma = k = 1$, Theorem 2.1 reduces to [1, Theorem 2.1, p.4].

Corollary 3.2
If $f \in \mathcal{K}_q(\gamma, \varphi)$, then

$$|b_{k+1}| \leq \frac{|\gamma| B_1}{2k},$$

$$|b_{2k+1}| \leq \frac{|\gamma|}{6k} \Big[B_1 + \max \Big\{ B_1, \frac{|\gamma|(k+3)}{4k} B_1^2 + |B_2| \Big\} \Big]$$

and for $\mu \in \mathbb{C}$,

$$|b_{2k+1} - \mu b_{k+1}^2| \leq \frac{|\gamma|}{6k} \Big[B_1 + \max \Big\{ B_1, \frac{|\gamma||k + 3(1-2\mu)|}{4k} B_1^2 + |B_2| \Big\} \Big].$$

Remark 3.3
For $\gamma = k = 1$, Corollary 3.2 reduces to [1, Theorem 2.4, p.7].

Remark 3.4
Taking $\lambda = 0$ and $\gamma = 1$, Theorem 2.1 coincides with [6, Theorem 2.2, p.620]. Also, for $\lambda = 0$ and $\gamma = k = 1$, Theorem 2.1 reduces to [1, Theorem 2.10, p.10].

Corollary 3.5
If $f \in \mathcal{P}_q(\gamma, \lambda, \varphi)$, then

$$|b_{k+1}| \le \frac{|\gamma| B_1}{k(1+\lambda)},$$

$$|b_{2k+1}| \le \frac{|\gamma|}{2k(1+2\lambda)}\left[B_1 + \max\left\{B_1, \frac{|\gamma||1+(1-k)2\lambda|}{k(1+\lambda)^2}B_1^2 + |B_2|\right\}\right]$$

and for $\mu \in \mathbb{C}$,

$$|b_{2k+1} - \mu b_{k+1}^2| \le \frac{|\gamma|}{2k(1+2\lambda)}\left[B_1 + \max\left\{B_1, \frac{|(1-2\mu)(1+2\lambda) - 2k\lambda|}{k(1+\lambda)^2}|\gamma| B_1^2 + |B_2|\right\}\right].$$

Corollary 3.6
If $f \in \mathcal{K}_q(\gamma, \lambda, \varphi)$, then

$$|b_{k+1}| \le \frac{|\gamma| B_1}{2k(1+\lambda)},$$

$$|b_{2k+1}| \le \frac{|\gamma|}{6k(1+2\lambda)}\left[B_1 + \max\left\{B_1, \frac{|\gamma||3(1+2\lambda) + k(1-6\lambda)|}{4k(1+\lambda)^2}B_1^2 + |B_2|\right\}\right]$$

and for $\mu \in \mathbb{C}$,

$$|b_{2k+1} - \mu b_{k+1}^2| \le \frac{|\gamma|}{6k(1+2\lambda)}\left[B_1 + \max\left\{B_1, \frac{|3(1-2\mu)(1+2\lambda) + k(1-6\lambda)|}{4k(1+\lambda)^2}|\gamma| B_1^2 + |B_2|\right\}\right].$$

Remark 3.7
For $\gamma = 1$ and $k = 1$, Corollary 3.6 corrects the results in [8, Theorem 2.1, p.195].

Remark 3.8
For $k = 1$, the results discussed in present paper coincide with the results obtained in [11].

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Nanjundan Magesh
Post-Graduate and Research Department of Mathematics
Government Arts College for Men
Krishnagiri 635001
Tamilnadu
India
E-mail: nmagi_2000@yahoo.co.in

Jagadeesan Yamini
Department of Mathematics
Government First Grade College
Vijayanagar, Bangalore-560104
Karnataka
India
E-mail: yaminibalaji@gmail.com

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