

# Phenomenology of the concept of tangent to a curve and related aspects of teaching mathematics

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## **Thesis summary**

The purpose of this study was to provide a multidimensional description of the concept of tangent and to capture various forms of its manifestation in mathematics teaching. In my research, I also looked for potential sources of difficulties in grasping the concept of tangent as experienced by mathematics learners.

The dissertation consists of seven chapters.

In **Chapter One**, I present selected elements of philosophical phenomenological analysis that can play an important role in the study of mathematical concepts. I also refer to examples of phenomenologically oriented studies in the field of didactics of mathematics, and I briefly discuss how I understand and to what extent I pursue the analysis of the concept of tangent in this dissertation.

**Chapter Two** of the thesis begins with a brief discussion of the problematic notion of a curve. I then refer to the etymology of the word *tangent* and introduce its various meanings in non-mathematical contexts. The main part of the chapter is an overview of the history of the development of the concept of tangent from ancient times to the 20th century. The analysis of different ways of defining the tangent throughout history provides a glimpse into how the perception and understanding of the concept evolved, and identifies moments when the meaning of the concept underwent significant changes. The juxtaposition of different definitions of the tangent proposed by contemporary mathematicians interested in the problems of teaching mathematics shows that the notion of tangent is still a subject of lively debate, and that the answer to the question of what a tangent is, is not at all as obvious as it might seem.

In **Chapter Three**, I first review the meaning of the term *concept image* and then provide an extensive review of educational research related to the image, definition, and various conceptions of the tangent concept. An extensive review of the literature allows us to determine what sort of educational problems related to the introduction and development of the concept

of tangent have been of interest to researchers thus far, and what difficulties related to the tangent have been diagnosed in learners of mathematics.

**Chapter Four** provides an overview of content related to the concept tangent that appears in the core curriculum for mathematics education at the secondary level, as well as an extensive analysis of how school textbooks introduce and develop the concept of tangent. The final part of the chapter also includes examples of alternative approaches to the tangent in school textbooks other than those currently used in Poland, and excerpts from several academic textbooks that shed new light on selected tangent-related issues addressed in high school.

The results and conclusions of the analyses were used to design and conduct a study of a group of prospective mathematics teachers on:

- the elements of their images of the concept of tangent and
- their skills in evaluating the correctness of an atypical solution of a typical task about determining the equation of the tangent to the graph of a polynomial function at a given point.

The empirical part of the thesis consists of two chapters. **Chapter Five** contains a description and the results of a written survey administered to 29 second-year undergraduate students (teaching specialization) majoring in mathematics at the Institute of Mathematics of the Pedagogical University of Krakow. The participants of the study were asked to:

- give the definition of a tangent,
- sketch tangents to the curves shown in whole or in part in the figures,
- determine equations of tangents to graphs of functions at given points, on the basis of formulas for the functions; and
- evaluate whether given sentences concerning the tangent are true.

In **Chapter Six**, I present the results of the second part of the study involving informal interviews with 14 students who, after completing the written part, agreed to participate in the continuation of the study. This time, the students were asked to analyze the solutions of two tasks concerning the determination of the equation of the tangent to the graph of a polynomial function at a point. The solutions were presented to the respondents as coming from a student. The university students played the role of teachers who were expected not only to judge whether each solution was correct, but also to prepare feedback they would give to the student.

**Chapter Seven** briefly summarizes findings from my research and analysis that are of particular relevance to mathematics teaching practice, and also suggests some possible directions for further research.