Line arrangements and symmetry groups

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Summary

The aim of this thesis is the investigation of the relation between ordinary power and symbolic power of an ideal in the polynomial ring of $n+1$ variables.

The dissertation consists of six chapters and one appendix.

In the first section we recall some basic definitions concerning the theory of hyperplane arrangements in projective spaces, and collect some informations about the so-called freeness arrangements. We give also a brief introduction to the symbolic power definition of an ideal in the polynomial ring of $n+1$ variables defined over the field $\mathbb{K}$.

The second chapter is devoted to properties of the simplicial line arrangements. The main results obtained in this chapter concern the lower bound of the number of intersection points of arrangements of this type. This outcome can be compared to the result of Bruijn-Erdős, which is mentioned at the end of the chapter.

In the next chapter we give the list of all counterexamples to the containment $I^{(3)} \subset I^2$ obtained for ideals I generated by some intersection points derived from simplicial arrangements. The main method of proving used in this chapter is the computer algebra system `Singular`.

The fourth chapter deals with inductively free case. We construct an infinite family $\mathcal{A}(12k+7)$ of inductively free line arrangements and we use it to give the negative answer for an open problem: *Are the containment* $(J(\mathcal{A}))^{(2r-1)} \subseteq (J(\mathcal{A}))^r$ *always satisfied for any* $r \geqslant 2$ *and any hyperplane arrangement that is inductively free?*

The next chapter is devoted to nearly free simplicial arrangements of lines. The main result of this part is classification theorem. We provide a complete classification of nearly free (and not free simultaneously) simplicial sporadic arrangements of $d \leqslant 27$ lines.

At the end of this thesis, we provide a catalogue of realizations of simplicial arrangements, from which we take certain points and create an ideal I which contradicts to $I^{(3)} \subset I^2$ relation. This list is a starting point for reconstruction of every configuration used in dissertation and can be used for a future investigation in this area.

An appendix consists of listings of three main procedures used during calculations made in `Singular`.