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AFFECTIVE TRANSGRESSION AND META-AFFECT: AN EXPLORATION OF PROCESSES FOR BELIEF CHANGE IN MATHEMATICS EDUCATION

Barbara Pieronkiewicz,

Pedagogical University of Krakow, Poland

barbara.pieronkiewicz@up.krakow.pl

Gerald A. Goldin

Rutgers University, USA

geraldgoldin@dimacs.rutgers.edu

Abstract: In underachieving students, well-established beliefs about mathematics or about their own ability may underlie response patterns of fear and/or aversion sometimes termed "math anxiety". Here we consider some possible processes of belief awareness and change in students who choose to address counterproductive beliefs voluntarily. We bring to bear two theoretical ideas: the concept of affective transgression (consciously crossing emotional boundaries established by prior beliefs); and the concept of meta-affect (affect about affect, affect about cognition about affect, and the monitoring and regulation of affect). In this paper we focus on processes of "getting over" affective obstacles, where students may confront their prior beliefs and accompanying emotions directly through new experiences that include changes in their meta-affect. We describe one student's processes of change. We suggest that teachers embrace the value of addressing affect explicitly within an emotionally safe teaching environment, and propose some ways to do so.

Go straight to the heart of danger, for there you will find safety.

– Old Chinese proverb

1. Mathematical beliefs and "math anxiety"

Aversion to mathematics, frequently termed "math anxiety," has long been widespread in students and adults (e.g., Tobias, Weissbrod, 1980; Ma, 1999; Baloğlu, Koçak, 2006; Varsho, Harrison, 2009; Ashcraft, 2002). Accompanying emotions may range from mild dislike or distaste to paralyzing anxiety or fear. Strongly-held beliefs are sometimes associated with such anxiety, including beliefs about the nature of mathematics (as complex, accessible only to those with unusual ability, and without much immediate relevance) and beliefs about oneself and one's (less than adequate) abilities in relation to mathematics (Leder, Pehkonen, Törner 2002; Maass, Schlöglmann, 2009). While considerable attention has been devoted to describing and measuring math anxiety, mathematical beliefs, and their association with learning and test performance, less has been focused on how best to approach attitude and belief change in mathematics education (Goldin 2007). Contradictory beliefs can be held without internal conflict, if in separated clusters; but people may also come to change beliefs perceived as incompatible (Op 't Eynde *et al*., 2002).

While beliefs can be highly stable, and belief change is typically time-consuming and exacting, Moscucci (2007) points out it is sometimes easier to "break off" a rigid element of the belief structure than an elastic one. Addressing well established beliefs in order to "break them off" is not easy, since every change in what one holds to be important and true involves some sort of disintegration, and thus a lack of the stability that may have been giving the student a sense of security (Greenberg, Rhodes 1991). Nonetheless, though it seems counter-intuitive, certain fixed beliefs may be more susceptible to change than apparently fickle emotional responses. There are notable examples of rapid and profound belief changes (e.g., Liljedahl, 2010). Even small, short-term social/psychological therapeutic interventions may have long-lasting effects on students' thoughts, feelings and beliefs, and (hopefully) "lead to large gains in student achievement and sharply reduce achievement gaps even months and years later" (Yeager, Walton, 2011, p.267).

How can we transform our methods of teaching so as to provide underachieving students with different, more powerful affect in relation to their interaction with mathematics? How can we do so in the face of already-developed emotional, cognitive, and social response patterns, including systems of belief that support their continued math anxiety? In this paper, we bring two key theoretical ideas to bear on these questions: the idea of *affective transgression*, and that of *meta-affect*. Here we provide a short explanatory account, exploring the relationship between these ideas as we discuss ways to approach students' affective barriers.

2. Affective transgression

The idea of affective transgression extends the thinking of Kozielecki (1987, 1997), who conceptualizes the human being as a self-directed, expansive creature developing through transgressive processes. His notion of *transgression* refers to the process of intentionally and deliberately exceeding physical, social or symbolic boundaries, including the crossing of personal boundaries, thus subverting the limitations that normally constrain our lives. According to Kozielecki, acts of transgression happen in four worlds: they can be taken toward (a) *material objects* – territorial expansion in the physical world, (b) *other peopl*e – expansion of control over others, or engaging in altruism and expanding individual freedom, (c) *symbols* – intellectual expansion, going beyond the information given, developing knowledge about the world, and (d) *oneself* – self-creation, self-development, unlocking and expanding one's potential, and coping with one's weaknesses. Transgressions may be of different kinds: psychological or historical, individual or collective, constructive or destructive, and so forth.

The term *transgression* has been adopted by Semadeni (2015), who defines a *mathematical cognitive transgression* as "crossing – by an individual or by a scientific community – of a previously non-transversable limit of own mathematical knowledge or of a previous barrier of deep-rooted convictions" (p. 27). Pieronkiewicz (2015), extending this perspective to the affective domain, characterizes *affective transgression in the learning of mathematics* as an intentional process of overcoming personal affective barriers that preclude one's mathematical growth and development. It can occur when one not only has some degree of insight into one's own affect, but also is willing to reflect on one's affect, and has the will (i.e. makes the choice) to bring about changes believed possible and good.

> In mathematics, perhaps because it is viewed as a difficult and important subject, students tend to internalize their experiences into their self-concept more than in other subject areas (Middleton, Spanias, 1999, p. 78).

Indeed, when a student has had consistently negative experiences in mathematics, the accompanying emotions can lead to the formation of beliefs and attitudes that endure. It is here that processes of affective transgression may enter constructively.

3. Meta-affect

Emotions may be of positive or negative valence, and systems of belief are often reinforced by powerful emotional feelings. But the emotions one feels *about* one's emotions, attitudes, and beliefs can transform the experience of them. Meta-affect (Goldin, 2002; DeBellis, Goldin, 2006) refers to affect about affect, affect about cognition about affect, and the monitoring and regulation of affect.

> In considering how individuals develop, we must note that prevailing belief structures in relation to mathematics are powerfully stabilized by meta-affect. Such beliefs are unlikely to change simply because factual warrants for alternate beliefs are offered (Goldin, 2002, p. 70).

Students may come to recognize some of their own traits, noticing their behavioral patterns, emotional reactions, and beliefs as they experience psychological limitations in doing mathematics. This recognition may be termed *meta-affective awareness*. The mathematical activity in this sense provides an opportunity for self-insight and for transcending barriers – an opportunity extending beyond the curriculum and the school setting.

The meta-affective awareness serves as a tool for intrapersonal (Heyd-Metzuyanim, Sfard, 2012) and interpersonal therapeutic communication, and a context for the student to prepare to cross previously-established boundaries. Undertaking a mathematical challenge (a transgressive act) may evoke familiar, negative emotions such as fear, frustration, guilt or anger; but new, emotionally safe meta-affective contexts encouraging curiosity, determination, self-respect, and pride can transform the students' experience of those negative emotions: e.g., frustration may be experienced with a sense of curiosity and challenge, or fear experienced with a thrill and a sense of pride in one's courage, leading toward mathematical accomplishment.

4. The supportive and therapeutic role of the teacher

It is important to raise the question of what kinds of knowledge teachers need in order to teach mathematics effectively. To provide help to students who have experienced mathematics painfully, teachers may find their own pedagogical content knowledge (Shulman, 1987) and mathematical knowledge for teaching (Ball, Lubienski, Mewborn, 2001; Hill et al., 2008; Silverman, Thompson, 2008) insufficient. That is, teachers may need to transgress their own previously-established boundaries of mathematical and pedagogical expertise, drawing on knowledge that belongs traditionally to other disciplines – psychology, counseling, and psychotherapy – to develop skills and competencies that go beyond traditional cognitive abilities and affective/motivational characteristics (Blömeke, Delaney, 2012).

Teachers taking such a therapeutic approach will establish *mathematical caring relations*, which include "a quality of interaction between a student and a mathematics teacher that conjoins affective and cognitive realms in the process of aiming for mathematical learning" (Hackenberg, 2010, p. 57). They will also need to have a deep understanding of what the student has been going through, and a concept of the method they are applying to assist the student's healing process. Kozielecki's descriptions of transgressive education and transgressive psychotherapy suggest that these have a common developmental goal:

> The strategic goal of transgressive education is to stimulate wide-ranging development of personality regulating human action – above all – transgressive acts (Kozielecki, 2001, p. 263).
>
> [Transgressive] therapy aims not only at restoring the health of a patient with some particular disorders (..) or solving his difficult life problem, but also at his development (Ibid., p. 245).

Likewise, the core objective of mathematics education in the case of anxious students should not be mere desensitization, reducing the intensity of the anxiety, nor should it be teaching the students how to solve mathematical tasks they find difficult. Rather, above all, it should stimulate the students' personal development and help them to find personal meanings (Vollstedt, 2009) or even "the meaning of it all" (Vinner, 2013) in the learning of mathematics. As Kozielecki (2001) states, "an essential means for achieving healing purposes is transgression" (p. 247).

5. Paths toward attitude and belief change

Affective transgression may take the form of *getting over* anxieties, fears and disempowering beliefs. The process involves confronting these obstacles consciously and directly through activity which – in the face of the negative emotions – leads to new emotional experiences, possibilities, outcomes and beliefs. This may be contrasted with a second approach, which may be termed *getting under* barriers. This involves consciously delving deeper into the affect and cognitions sustaining math anxiety, finding their roots in the person's intimately-held history, understanding the tangible causal effects that past experiences and present unconscious desires may have on current feelings, interpretations and behaviour, and finally rebuilding new, more powerful and ultimately more satisfying affect.

Both approaches presuppose a conscious, personal decision to pursue change by the student in the intervention. Making such a choice, however, requires the student to have some degree of meta-affective awareness.

In our characterization of a process of change in the affect of a low-achieving student, the immediate incentive to challenge the status quo may originate with the external facilitator. Yet to accomplish personal transgression, the student must first acknowledge the existence of some limitations and boundaries to be extended. We suggest that in the process of affective transgression, "getting over" barriers may be modeled as a relatively short-term intervention involving four successive phases: Decision, Exposure, Experience, and Post-reflection. Of course such an intervention will not always succeed, but sometimes it can have dramatic positive effects.

5.1. Decision

As the first phase is crucial to the whole process of affective transgression, we devote to it our greatest attention, describing some possible obstacles to the student's making a positive decision. During the Decision phase a student, encouraged by a teacher or an external facilitator or on her own, agrees to face what she fears or has sought to avoid. The student expresses readiness to take a certain *risk* by relating to difficult or painful past experiences. The discussion of affect with the student becomes overt, and a powerful meta-affective context is set in which the student sees the possibility of and the potential benefits in having a different orientation toward mathematics. The major driving forces in the Decision phase are *discomfort* and *hope*: discomfort caused by the discrepancy between "who I am" and "who I wish to be," and hope that the envisioned change is possible. According to the neurological findings of Lyons and Beilock (2012), in math-anxious students the *anticipation* of an upcoming math problem activates regions of the brain responsible for detecting threats of harm and physical pain. Due to the intensity of affective arousal, the Decision phase may be emotionally the most difficult to handle. We see this phase not as a single act, but as a complex process influenced by many factors.

The student's past experiences can either support or hinder her decision. They provide a frame of reference, on the basis of which she evaluates the likelihood of success. In the case of a "math anxious" student, repeated negative experiences of mathematical activity – including possibly the emotional pain of failure, embarrassment, humiliation, and their consequences for the student's perceived self-efficacy and self-esteem – may lead to the formulation of beliefs serving as tailor-made protective armor (Pieronkiewicz, 2015). The student may come to believe that mathematics is useless, and not needed in her life, or that to succeed with mathematics one needs to have a "mathematical mind." Thus someone's belief that she has "humanistic predispositions" can serve as a justification and explanation of the fact that she is not a successful math learner.

Such beliefs reduce discomfort and ease the pain when a student has been attempting to achieve some particular goal but failed. Nimier (1993, p. 30–33) speaks of such reactions in terms of *defense mechanisms* students employ against mathematics in order to cope with anxiety. They may take the form of phobic defenses; e.g., avoidance ('Doing maths is doing something which to me seems impossible'), repression ('Doing maths, it represents nothing, it's absurd'), and projection ('In mathematics there is no place for personality; all that you do has been done before, everything has been seen already', 'Those who do too much mathematics sometimes risk not having your feet on the ground'). In the Decision phase these beliefs/defenses are being challenged, possibly for the first time. They will be fighting back during the latter stages of the healing process of affective transgression.

A person who constantly experiences his own shortcomings and failures may come to believe that he is doomed and no matter what effort he makes, he will encounter another failure anyway. In this frame of mind, he has no influence and control over external events. He internalizes expectancy of failure and that leads to *learned helplessness*, sometimes manifesting itself as anxiety (Seligman, 1975).

> Thinking that bad things are going to happen no matter what you do (the learned helplessness response) is not likely to lead to greater persistence,

> whereas expecting positive outcomes can increase the motivation to try and persist (Peterson, Seligman, 2004, p. 232).

From the perspective of Kozielecki, what may be important in overcoming the vicious cycle of learned helplessness is *hope*, defined as a "multidimensional cognitive structure, of which a core component is the belief that in the future one will be offered the good (achieve an important objective), with a certain degree of certainty, that is with a certain probability" (Kozielecki, 2006, p. 37). Undoubtedly, hope greatly contributes to the process of decision making. To lead to a positive decision, however, the strength of the hope must be stronger than the fear of change.

Highly math-anxious students' resistance to change may be, at least partially, attributable to more general reasons why people are afraid of change. There are at least three such reasons. First, change means stepping out of one's comfort zone, challenging the secure equilibrium state and leaving what one is familiar with, to embrace the unknown. Thus it is a call to action, and requires taking responsibility for the outcomes. Secondly, change is rarely confined to a single, narrow range. Even minor changes may, as in the "butterfly effect", entail a series of hard-to-predict further changes for which the person may not be ready. And finally, the resistance to change may come from the dread of losing one's sense of identity and coherence. Deep affective changes shake one's inner world and call into question the integrated image of self-organization. Thus choosing to transgress one's affective limitations requires courage and determination.

> A stable sense of self is not dependent on stable content but rather on a sense of the self as an agent with continuity over time and as possessing coherence as an autonomous existence. (...) A person is not dependent on a stable form for a sense of identity but rather on a sense of self as the agent of experience with continuity over time" (Greenberg, Rhodes, 1991, p. 43)

We believe that what is at risk of being shattered is a static and rigid sense of the self. This is a self that has grown tolerant of and familiar with needs not being met but is too frightened to risk further disappointment" (Ibid., p. 54).

5.2. Exposure

The Exposure phase involves *openness* to the possibilities inherent in new experiences, and thus also *vulnerability*. Meta-affective contexts accompanying Exposure include courage, determination, and pride (*hubris)* in addressing something personally difficult, allowing fear or anxiety when they occur to be experienced differently. The idea that "it is not that math itself hurts; rather, the anticipation of math is painful" (Lyons, Beilock, 2012) suggests addressing emotional control *prior* to mathematical activity (Beilock, Willingham, 2014), which is accomplished in this phase.

In letting herself be exposed to what has previously been threatening to her, a student open herself up before the teacher. Such openness, with affective components that include fear, vulnerability and trust, is a transgression toward the other person – the teacher. It requires emotional effort, and overcoming adhesion to self-protective behaviours:

> This struggle is understandable, for participating always involves risk. If one goes out too far, one will lose one's identity. But if he is so afraid of losing his own conflicted center—which at least has made possible some partial integration and meaning in his experience—that he refuses to go out at all but holds back in rigidity and lives in narrowed and shrunken world space, his growth and development are blocked (May, 1983, p. 20)

It is of the utmost importance in this phase not to expose the student to something that exceeds her limits. The teacher has to be cautious, empathetic and understanding. Rather than forcing a student to proceed the teacher has to monitor the student's reactions to each step.

The purpose of the exposure phase is to guide the student toward *corrective emotional experience* (Alexander, French, 1946) in the context of mathematical activity. The therapeutic principle of this method as it is used in psychotherapy is

> to re-expose the patient, under more favorable circumstances, to emotional situations which he could not handle in the past. The patient, in order to be helped, must undergo a corrective emotional experience suitable to repair the traumatic influence of previous experiences (Alexander, French, 1946, p. 66).

5.3. Experience

In the Experience phase, the student actively takes up a mathematical challenge. This is a highly transgressive act: engaging with what the student had previously been reluctant or afraid to engage in. The teacher's role now is to create an emotionally safe environment, one of sustained trust in which – under favorable circumstances – the student can manage emotional experiences which previously would have been scarcely bearable. The major purpose of this phase, however, is "*not* to eliminate frustration, remove fear and anxiety, or make mathematical activity consistently easy and fun" (DeBellis, Goldin, 2006, pp. 136–137), but to develop the student's meta-affective competency, transforming negative emotion or difficulty into potentially productive experience. The emotional feelings that were previously unbearable can here be experienced not merely as manageable, but as interesting and intriguing. The student experiences a sense of fulfillment associated with the context created during the Exposure phase: The first two phases have led to the declaration, "I will do this," and now the third leads to the experience, "I did it!" accompanied by relief, pride, and satisfaction.

5.4. Post-reflection

Essential to the creation of longer-term, powerful affect is the final, Post-reflection phase. This is where lies “the secret of the therapeutic value” (Alexander, French, 1946) of affective transgression. Here the conclusion becomes explicit to students that “the consequences were not nearly as bad as they anticipated – and, indeed, might even be quite favorable” (Goldfried, 1991, p. 34). The dissonance between earlier, predominantly negative associations with mathematical activity and current, positive emotional (and meta-affective) experience is highlighted. The old tension and fear have recurred as in earlier, similar settings, but because of the significantly different orientation of the student and attitude of the teacher, the student may have experienced the “old problem” in a very new, healing way. This enables the student to reconsider her views of mathematics and of herself as a mathematics learner – which also is a transgressive step, involving some disintegration of prior belief structures and construction of new ones. We suggest that for some students, even short term social-psychological, therapeutic interventions pursuing this pattern can have long-lasting effects on their thoughts, feelings and beliefs (Yeager, Walton, 2011).

In describing psychotherapy, Greenberg and Rhodes (1991) suggest:

> Therapy, rather than attempting to repair the self, reengages the self in the process of self-organizing. New experience in therapy is a change in the manner in which the person is organizing him- or herself (p. 44).

This characterization captures the therapeutic dimension of the process of affective transgression, essentially a therapeutic experience in the learning of mathematics that empowers a student to reestablish control over her mathematics-related affect.

6. An illustrative case: Sara

An approach based on the above-mentioned phases may be considered as an intervention in a classroom setting, in a small “remedial” group, or in an individual tutorial. To illustrate, we provide a highly abbreviated report about “Sara” [not her real name], a high school student in Poland.

6.1. Method

At the time of the intervention, Sara was 18 years old and in her final year of high school in Poland. The first author worked as a tutor with Sara individually, meeting (on average) weekly for one to two hours over a period of four consecutive months. As a tutor and reflective observer, she created detailed “field notes” after each class she conducted with Sara. She also administered a retrospective questionnaire seven months after the last tutorial. The description of what occurred is drawn from this collection of notes together with Sara’s response to retrospective questions.

By way of background, in the Polish educational system secondary school graduates take a national examination. The examination is compulsory only for students who want to continue their education at the tertiary level. When Sara was in her final year of high school, this exam consisted of a mandatory part at a basic level (Polish language, a foreign language, and basic mathematics), and additionally up to six subjects at an advanced level. The criterion for passing was for the student to receive at least 30% of the points available in each mandatory subject. Since 2010, when the basic exam in mathematics became compulsory, most of those who did not pass had been students who failed in mathematics. Sara wanted a mathematics tutor because she was about to take the high school final exam in mathematics at the end of the school year.

6.2. Results, observations and interpretations

The first meeting with Sara began with her declaration that she was a "humanist," and wanted only to pass the math exam at the basic level. She was aware of some gaps in her mathematical knowledge, and said she had problems with understanding mathematics. At the beginning of the first session, Sara seemed fraught and cagey. She was hoping to find in her new tutor somebody who could help her understand mathematics, but said she did not truly believe she could make any significant progress. Doing mathematics had been unpleasant for her so far, and each time she got stuck while solving a problem, she was more inclined to give up than she was to put in more effort or try a different strategy.

During the first month's meetings, the tutor identified two contradictory beliefs in Sara. She held the profound conviction that she had a "humanistic mind," which served to *justify* her mathematical difficulties and brought a temporary sense of alleviation of responsibility. But she also *blamed* herself for being mathematically handicapped, feeling accompanying guilt. A few weeks later, when Sara described the environment in which she grew up, the tutor concluded that she had not been raised to appreciate effort and constructive struggle. Rather she had been told that problems usually have external sources, and can be solved by finding somebody who could resolve them for her. Thus, Sara entered the tutoring sessions with an expectation that from that point on, the tutor would be responsible for her mathematical growth. She gave the impression of somebody proud, self-confident, and slightly condescending.

Decision. From the very beginning, the tutor encouraged Sara to face her problems, and to see that she herself was capable of solving them. During the first meeting, Sara also questioned the importance and meaningfulness of learning mathematics, asking explicitly what a good reason for learning it would be. The tutor replied that she could only share her own reasons, and advised Sara to consider their usefulness and adequacy for herself. She introduced the idea of affective transgression, including overcoming one's limitations, facing what at first sight seems to be frightening, and taking on the challenge of trying a problem that seems difficult to solve. The advantages of this *transgressive* approach, the tutor said, could go far beyond the boundaries of mathematics to serve other, non-mathematical purposes by the means of non-specific transfer.

Sara responded that she liked that idea, and expressed real astonishment that nobody has ever told her anything like this about mathematics. The tutor's reply surprised Sara because it linked mathematics to Sara's own personal development, rather than to practical applications of mathematics in daily life or to the need for passing the final exam. We may (only) conjecture that Sara asked the question about a reason to learn mathematics in order to "test" the tutor, to trap her into a no-win situation. This exchange between Sara and the tutor turned out to be an ice-breaking moment, in a way that was unexpected by the tutor. It was surprising that such a critical point in the tutorial appeared so soon – quite an unusual experience for the tutor, as she knew that such moments do not normally happen so early, if ever.

We remark that some teachers may find students' questions regarding good reasons for learning mathematics to be irritating, and interpret such questions as signs of resistance. We propose the value of seeing them instead as students' *call for meaning*; and we advocate considering each such occasion as an opportunity for deeper conversation with the students:

> A number of studies demonstrate that students are more motivated when they possess a "purpose for learning" — when they understand how learning today will help them accomplish meaningful goals in the future. (Yeager et al. 2014, p.559)

What seems to be important in responding to the question, "Why should I learn mathematics?" is anchoring the answer in values that hold deep, personal meaning (Vollstedt, 2009) for the student. Human beings respond to value with their will (Wojtyla, 1985), and are more eager to take responsibility for what they truly value.

Exposure. After the event described Sara became more open, allowing herself to be exposed to problems that previously had exceeded her abilities. But sometimes emotions related to Sara's non-mathematical experiences precluded her from working effectively. During the third meeting with the tutor, Sara appeared distracted, very sad, and unable to focus on the mathematical content. The tutor stopped the lesson and asked Sara what was bothering her. Sara then shared some intimate details about her relationship with a boyfriend who had just broken up with her, and a close friend who had also unexpectedly ended their friendship. As she was leaving the session, Sara thanked the tutor for letting her share her feelings. She felt much better. Subsequently she seemed to become more trustful, more willing to talk openly about the true feelings she had been experiencing while solving mathematical problems.

We conjecture that as the tutor (transgressively) welcomed Sara's personal life issues into the time normally dedicated to mathematical conversations, Sara reciprocated by allowing mathematics into her personal life. In that moment of openness, when Sara experienced being noticed by and important to the tutor, she became able to express feelings without humiliation. The tutor did not judge her for inappropriate behaviour, and Sara's meta-affect toward her own emotions in doing mathematics changed profoundly. In the next session Sara confessed, "You were right. Mathematics is very helpful. Now, when I feel sad, I start solving some math problems and that cheers me up."

Experience. The most remarkable change observed during the remaining tutorial sessions was that although Sara still experienced many difficulties while solving problems, she became involved and eager to spend more time struggling. She started believing that after receiving some help from the tutor, she would be capable of fully understanding the school material. Even small successes inspired Sara to put more effort into doing mathematics. On the days separating consecutive meetings with the tutor, she was solving (or trying to solve) not only the assigned problems, but also additional ones that she herself had chosen.

Post-reflection. During this time, episodes of Post-reflection were interspersed with Sara's successful problem-solving experiences. She was encouraged to *notice* her transgression of her earlier boundaries: her successes, her belief changes and her new ways of feeling about mathematics.

Seven months after Sara completed her tutorial, her tutor (the first author) asked her to respond to a written retrospective questionnaire. We wanted to learn how she recalled the tutorial experience, and whether the apparent changes in her affect around mathematics had persisted and had any longer-term impact on her life.

Sara was asked why she had been willing to learn mathematics with that particular tutor. She responded [translated by the first author from the Polish]:

> You were calm. The fact that I was able to ask any question, not being afraid that I would be laughed at, because, for example, I was asking about something trivial or basic or something I should have already known. If I didn't know something, I was just asking and you were explaining everything to me. For sure you were also motivating me in the moments of doubt (…). I guess, you were the first person, who didn't treat me like a mathematical imbecile, but rather like a person who is a form that can be shaped, and can learn mathematics ☺ It is a pity that I came to you so late,

> because it would be nice to prove to me and my brother that I could be good in mathematics.

The phrase, "a person who is a form that can be shaped" suggests that behind her initial proud and confident mask, Sara was hiding her very vulnerable nature. Contrary to the popular equating of a humanistic orientation with mathematical disability, Sara – defining herself as a humanist – discovered the beauty of mathematics and some humanistic aspects of mathematical activity. Her response also suggests high aspirations. In acknowledging a potential that perhaps nobody else saw in her, she now considers her previous successes unfulfilling: she knows she could do better. When asked about memories from her school years, Sara focused mainly on her teachers:

> For a year or two (grade 4–5) I had a rather not nice old teacher. Later, I'm sure it was in the 6th grade, they changed our teacher. The woman who became our next teacher was the best math teacher I have ever had in school, and in general she was the best teacher ever. She taught my brother too, and he remembers her the same way I do. It was evident she liked kids and she also liked her job. She wanted to share her knowledge without that overwhelming fear during the classes. The fear about math disappeared, and math classes were nice. Nevertheless, even in primary school I had some private lessons. From time to time, but still.

She was not asked directly about her parents' interest in her education, but Sara wrote:

> I think that in primary school these are the parents who help the children with lessons, but mine didn't have time, so they provided me with additional classes. At home I've always been told that I had a humanistic and artistic mind and since my primary school I didn't even consider the thought that I could ever be good in mathematics.

In setting out to overcome the fear of math and a strongly held conviction that she was mathematically incapable, Sara transgressed not only her affective limitations, but stood against who she had been told she was supposed to be. In Gymnasium [prior to high school] she often experienced math anxiety, and her low self-esteem was confirmed:

> The time of Gymnasium was the worst and I would rather not go back to this. My math teacher was the worst teacher I ever had, taking into account not only mathematics but all subjects in general. I remember, that I was sitting in the last school desk … because I wanted to be as far as possible from that teacher. When entering the classroom I was paralyzed with fear. Then I began to have private lessons regularly. The tutor was wise, I could have asked him about anything with no embarrassment, but unfortunately he didn't know how to teach and he was often just solving problems for me.

However, when asked about her most negative math related memory from school, Sara gave an example of a situation that took place in high school:

> (…) our teacher told my class that we would not pass the final exam anyway, and we should surely meet again in a year.

As her most positive memory, she cited effects of the tutorial, when she felt:

> (…) pride that I was able to solve problems. Left to myself I started doing some tests from the book and I was solving problems for pleasure.

Sara was asked to evaluate (on a scale of 1 to 10) how math anxious she was as a school student. She recognized what she was truly afraid of, while saying she was afraid of mathematics:

> Primary school: 2 (only grades 4–5) (…) Gymnasium: 10. Neither before, nor after was I so afraid before the classes. (…) High school: 4. (…)
>
> I just realized that I rate the fear about mathematics basing on rating how afraid of my math teachers I was in each type of school. That was not fear about mathematics, but rather about math teachers. After all, math is just a school subject. And now I know, everyone is capable of learning math, you just need to work on yourself.

Then we asked whether the meetings with her math tutor changed anything in Sara's attitude toward mathematics:

> They changed a lot, because they helped me believe, that yet I'm able to learn it and this is not something for the chosen ones. I even think, that if I have met you earlier, I would be quite good in math. It is a pity, that for 12 years of my education everybody was telling me I was a humanistic mind, and that I was hopeless in math. Only 4 months before my final exams I realized, that you can learn everything if someone is sympathetic to you, is willing [to help you] and knows how to explain.

The last question posed to Sara was the following: "Students have different reasons for learning mathematics. When they ask me about reasons for learning mathematics, I tell them about overcoming one's limitations, victory over one's fear, entering the struggle and facing difficulties instead of escaping from the problem. How do you find these arguments? Did you have such experiences in our classes?".

Sara replied:

> I remember I asked you this question – 'Why should anybody learn mathematics' – and you told me that it develops, teaches logical thinking and wouldn't it be nice to prove to myself, that I can do it, instead of hiding behind the mask of "I'm not good in math". You know, this gave me a lot, really. Now, I'm learning Japanese, and at the beginning it was sometimes hard, but in such moments I was thinking that, after all, others have learned it, why should I be worse? Why do I always give up, instead of knuckling down and overcoming my own limitations? I've been learning Japanese for 5 months now, and I'm still getting better. This all started in your classes.

The changes Sara underwent seem to have had long-term effects. We can infer that some new, more powerful affective structures have developed, that she transfers readily from the domain of mathematics to the learning of Japanese, and presumably to other domains as well. This corresponds to an objective of mathematics education highlighted by Krygowska (1986). She refers to attitudes and intellectual behaviors functioning beyond mathematical activity, and being developed through *transfer* and the adaptation of/adjusting of attitudes and behaviors specific to mathematics to other domains of human activity. The interesting problem of such *affective transfer*, which is by definition transgressive, appears not to have yet been extensively researched.

The whole process of Sara's challenging her affective limitations resulted in several remarkable changes in her beliefs, attitudes and meta-affective competences. In response to retrospective questions, we received information that rendered explicit some affective changes and their long-lasting character. We summarize these changes in Table 1.

Before	**After**
fear	not being afraid
afraid of math	*math is just a school subject*
mathematical imbecile	*a person who is a form that can be shaped and can learn mathematics*
humanistic mind not capable of doing mathematics	*someone who can be good in mathematics*
mathematics for the chosen ones	*everyone is capable of learning math*
just to pass the exam	prove to herself and her brother … *It's a pity we haven't met earlier…*
externally solved problems	*you just need to work on* ***yourself*** (responsibility)
letting go	overcoming limitations

Table 1: Changes evidenced in Sara

7. Conclusion and limitations

We have discussed a possible process of belief change in math-anxious students involving four phases: Decision, Exposure, Experience, and Post-reflection. By means of a conscious choice that entails affective transgression, with the support of a teacher or tutor, a new meta-affective context is created, allowing the student to "get over" her anxiety and associated defenses, and experience mathematics in a new way. Our discussion draws on the work of many psychologists and mathematics educators. The process of change in Sara, described briefly, is intended to illustrate and help elucidate the discussion in the paper: the dissonance in the student's initial beliefs, the phases in the process of change she underwent, the importance of her conscious, transgressive decision, the vulnerable exposure, the experience of success, and the conscious attention to meta-affect before, during, and after each phase.

But the model is highly tentative, and not the only approach to overcoming math anxiety. The story of Sara is at best merely illustrative and suggestive, and involves much interpretation and inference. The extent of her transformation is unusual, and of course it cannot be a basis for any broadly generalizable conclusions. We would like to see considerably more research on the affective aspects of students' mathematical belief change.

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