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USING THE SEMIOTIC POTENTIAL OF VIRTUAL ARTEFACTS IN DEVELOPING ALGEBRAIC THINKING

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Abstract: The main goal of the research I report here was to determine whether a special kind of tool like a computer game, can help children to learn how to operate on algebraic symbols and also how to solve linear equations. I run an experiment in the classroom, with a group of twenty 12-year old children, who were using a video game (DragonBox Algebra 5+, 12+, 2012–2013) during algebra lessons. While playing the game they discovered algebraic symbols and operations. Adopting the theory of Mariotti and Bartolini Bussi (2008), I used DragonBox as a semiotic tool. The results of this study show that such an artefact as DragonBox may serve as a source of many mathematics meanings.

1. Introduction

One of the major goals of teaching basic algebra in primary and middle school is to provide students with skills that would enable them to use algebraic language necessary for formulating rules, justifying statements, proving theorems and solving problems. Unfortunately, the results of the traditional instruction are far below our expectations: many students do not use algebraic notation correctly and find it difficult to transform expressions or solve equations, even the very simple ones. In the reports of the Central Examination Committee, summarizing every year examination for middle schools students, we find commentaries like these: "*It's very difficult for students to use algebraic expressions for describing different connections...*" *(*Osiągnięcia uczniów kończących gimnazjum w roku 2012)*; "…students had problems with solving equations, they made mistakes in transforming algebraic expressions...*" (Osiągnięcia uczniów kończących gimnazjum w roku 2015). There are many reasons for such situation. Actually there is not enough time for mastering algebraic skills in the classroom. Students have to learn many different rules in short time and most of them fail to do so. On the other hand teachers rarely take the advantage of using non-traditional tools that could facilitate the process of learning algebra. I surveyed 50 teachers of mathematics and most of them said, they had not ever used anything except the balance metaphor for solving linear equations. So algebra seems to be very formal from early beginning, though, for many years, besides traditional artefacts used at school, we have been offered many new ones, connected with computer technology. Researchers agree, that such tools can open new ways for effective teaching.

2. Student's difficulties with solving equations

There are some basic conceptual obstacles in learning algebra in the specific area, named by Malara and Navarra (2002) as „pre – algebraic field". Many of these spring up from unexpected arithmetical contexts. The authors point out that inadequacy of arithmetic comprehension hinders the **transition from arithmetic to algebra.** They also highlight the **differences between natural and formal language**:

> It is believed that unconscious habits and cognitive process – specific for a natural language – may create conflict with the procedures required from a formal language (p. 229).

They give an example:

> *"y is three times bigger than z"* is literally translated erroneously as „$y = 3x + z$" or „$y = 3x > z$".

Difficulties with learning algebra often come from **erroneous understanding of the concept of algebraic expression** (Turnau, 1990). Students do not know, that it has got at least four meanings: as a scheme of calculation, as a name of the number, as a variable, or as a symbolic structure.

Zaremba (2004*)* addresses the problems with **the sign of equality** in learning arithmetic. She gives an example of incorrect use of the symbol in notations such as: $27 – 7 + 3 = 27 – 7 = 20 + 3 = 23$. The only sign used properly is the last one. There are plenty of such lapses in students' notebooks. The sign of arithmetical operation is such a strong stimulus for many children, that they want to perform operation shortly after they have accomplished the previous one.

As Pirie and Martin (1997) say, children have **to change their understanding of equality sign when they start solving equations**. Till this moment it is sufficient to see it as a result of the operations. You can see equations like this: 2x – 3 = 5 in the sense: I had got something, but someone took away three things from me, and now I only have 5 of items. The situation has its own rational order. But if you move on to the equation: 5 = 2x – 3, the order of the action is reversed, the result is before action. To many students it does not make sense, because they can only read equations from the left to the right side, as in the former example. It is therefore important, to perceive the equation as a whole – static entity, not as a temporal sequence of action. Zawadowski (1996) considers these difficulties as language problems:

> It's very hard to give the meaning to the simplest equation $x = 3$, because we have x on the left (variable) and 3 on the right (number), so how to understand they are equal? (p. 142).

Filloy and Rojano (1989) pay attention to the **difference between arithmetic and algebraic thinking**. It concerns not only the interpretation of letters, understanding of the sign of equality or the habits of noting the operations. It also requires completely new skills. Algebraic knowledge has to be constructed arithmetically but there comes a moment when you have to go beyond arithmetic. The authors define this moment as a **"didactic cut".** The equation $Ax + B = C$ can be solved by arithmetic operations, but you cannot do the same with $Ax + B = Cx + D$. You must perform algebraic operations on variables to solve the second equation. And the transfer cannot be spontaneous, it has to be carefully planned by teachers, and built on specific modelling, to make the children become able to operate on new algebraic objects.

The most common method of introduction into the linear equations is 'the equation as a balance'. Pirie and Martin (1997) ask the question – how real is the image of the balance for modern students? Most students have never seen such an apparatus and do not understand the physical activity of weighing in this way. Moreover, solving balance problem, we get the answer as a weight, a quantity, not simply a number. What about equations with negative solutions? Other models cause other troubles. As Filloy and Rojano (1989*)* state, it is very important to remember, that there are **two aspects of modelling: translation** and **separation**. The first one touches the transition from the real situation to abstract level; the second – takes place when we separate objects and operations from concrete model. These two must not disturb each other.

3. Why computer games?

Contemporary research on using digital technology in education show, that **computer games can effectively reinforce instruction**. Games give opportunity for engagement (Shute, Ventura, 2013), help to master specific skills, open up the possibility of interactive and decision-making based contexts of learning (McAlister, Charles, 2004). Moreover, computer games can create a new style of learning, which is more familiar to modern students (Prensky, 2001). There are also some research that confirm the use of games develops spatial imagination (Green, Bavelier, 2007) and may positively affect concentration of attention (Shaw, Grayson, Lewis, 2005). In Devlin's (2011) opinion, a good game can help students to jump over the „symbol barrier" in learning algebra. He compares mathematics to music, which is written in notes, but its essence stays somewhere deeper. Due to technology children can do mathematics like they learn to play the piano.

4. Semiotic potential of artefacts

As Mariotti and Bartolini Bussi (2008) state, people the word „artefact" may refer to many different things like sounds, gestures, tools, books, instruments, information technology tools. Creating and using artefacts for educational purposes is quite common, but what lies at the heart of artefacts application is the ability of transferring their influence to the cognitive level. According to Norman (1993), „cognitive artefacts" have double nature: they are oriented outwards – that is, they can change the environment, but also they are oriented inwards – since they can influence on someone's mind. Mariotti and Bartolini Bussi (2008) define so-called ***semiotic potential* of cognitive artifact** saying, that it can be the source of two kinds of meanings – very personal meanings and a mathematical ones. They emphasize the role of the teacher in this process, which cannot be random but has to meet special criteria. There are three kinds of symbols, that children discover on their way from using artifacts to operating within mathematical context: *artifact signs, pivot signs, and mathematical signs*.

5. Research description

The game, that I used during my experiment (DragonBox Algebra 5+, 12+, 2012–2013), is widely scrutinized on websites. It is highly praised for giving opportunity to enjoy difficult algebra (Fukumoto, 2016). Other articles emphasize, that children can discover mathematics using the game, and everyone can be actively engaged at his or her own level (Teaching Algebra: DragonBox as a Resource, 2015). Moreover, Dragon Box gives alternative tools for learning for modern society (Bridges, 2014). On the other hand you can find remarks, that playing the game does not make children solve real equations, because they only manipulate with the symbols on the screen, without understanding (Trausan-Matu, Boyer, Crosby, Panourgia, 2014).

Research question: Can a specially chosen tool (cognitive artefact) help to overcome the difficulties experienced by school students in learning algebra calculations? What obstacles can students jump over using it?

Research tools: Video game (DragonBox Algebra 5+, 12+, 2012–2013); PC's, tablets, interactive whiteboard, mathematics school books.

Research group: Experimental group of twenty 12-year old pupils; control group of 80 students (14–16 year old) attending to classes where I was teaching mathematics.

Course of research: 10 lessons based on the use of computer game Dragon Box for solving linear equations with one unknown.

Data collection: observation and conversation with students during the lesson; recording the discussions that the students had when working in pairs; interviews with students correcting their errors; analysis of students' written solutions; analysis of the results of a test and teachers' survey responses analysis.

Analysis of research results: I compared the experimental and control groups for solving equations. I analysed the procedures used by all the students, the errors they made and I also distinguished solution strategies which were specific to experimental group.

6. Dragon Box

The game begins with a presentation of the table divided into two parts and different types of cards. There is one particular card among them – a blinking box. The main principle says:

In order to win you must isolate the box on one side.

Students follow the rules that say what move is needed to get rid of the useless cards – what to do if the cards are scattered, if they are stuck together, or if one is below another. One of the first rules says:

You can add the card from the deck.

From now on the students always get this information with a leaping picture on the deck of the board. They cannot make the next move until they place the same card on the other side.

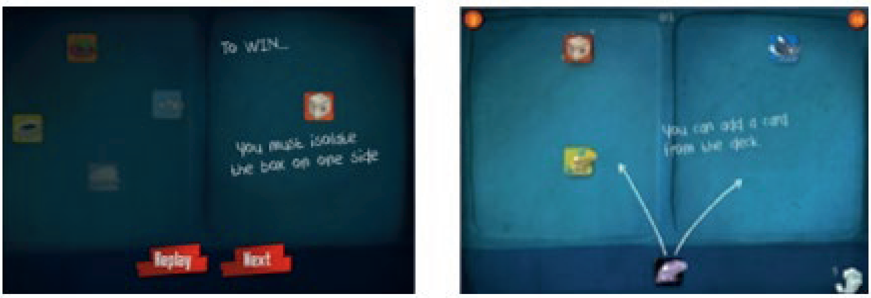


Figure 1: The main rules

Next principles come in slowly and they are used in several examples before anything new is introduced. Having solved an equation the student gets feedback. You can always move back. You can also start solving your task again from the beginning. There is no timing in the game - students have as much time as they need to finish. The game starts from replacing color icons but later on during the game the pictures are replaced by cards with numbers and letters. Soon the "blinking box" is transformed into the card with "x" . On one of the last levels, the signs of arithmetic operations appear and the line dividing the board is replaced by the equal sign.

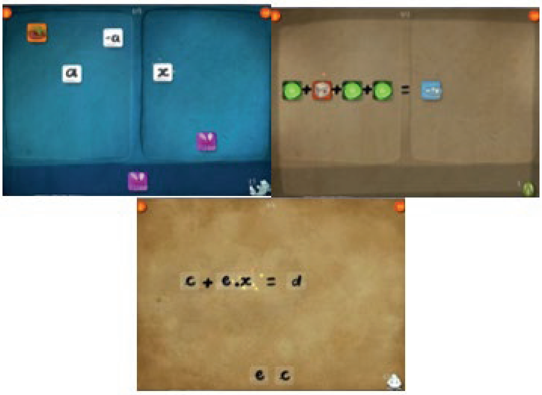


Figure 2: Subsequent levels of the game

During the first two lessons students had opportunity to play, discuss every example and look for the best strategy. They often moved back the steps, solving the task from the beginning. Some students cooperated, others worked without any help. I acted rather like an observer – sometimes I helped students to understand English commands. I was not interrupting and I was making no suggestions. I was listening to my students and watching what they had discovered. How did they link the pictures with numbers and the moves with arithmetic rules? In more difficult examples students thought about the order of the moves. They noticed that it is very important to see where the box is at the beginning, to decide about the order of the moves.

Students made a lot of mistakes and moved back many times. I can say – they learned a lot from their mistakes. After getting the feedback they knew what was wrong and what was correct, they knew if the order was the best and if there were no useless cards.

To make the transfer to „paper-and-pencil work" easier, students started from the easiest equations. I encouraged them to code all needed operations in such a way, that would be legible and understood. Pupils created notation for: adding cards from the deck, moving one on the other, dragging cards. Ideas were very rich, and **first „artefact signs"** appeared:

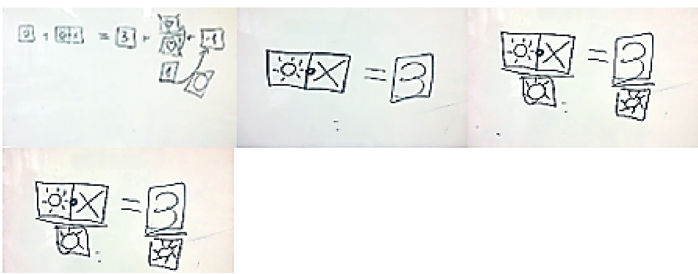


Figure 3: Artefact signs

Student’s commentary:

> I put Heart on the Heart, One on minus One and I get Zero. Then, I want to get rid of the Sun on the left side, so I put the Sun under the left and right side. Eventually, I put Sun on the Sun and get One.

Then, after some iconic examples, students began using mathematical symbols, but they were still speaking the „game language”. Sometimes, new words appeared (**pivot signs**):

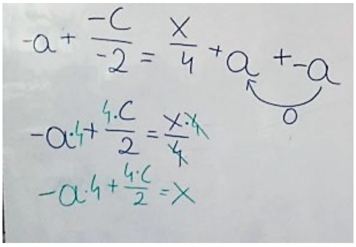


Figure 4: Pivot signs

Student’s commentary:

> To get rid of Four from the bottom on the right side, I must put the same to the top, or stick Four to every group – I mean – multiply by four.

Very slowly students started using the **specific algebraic language** instead of the language of the game. But they could always adduce to Dragon Box. Particularly in more difficult, doubtful situations, coming back to the game helped them to make the right decision.

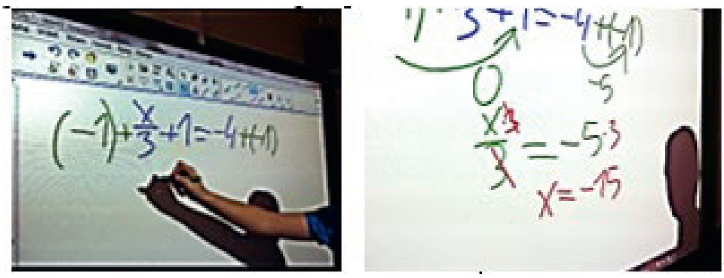


Figure 5: Specific algebraic symbols

Observing children working with the DB, you can see the way, that they come along from the words and signs specific to the game, to formal symbols, due to the semiotic mediation (Mariotti , Bartolini Bussi, 2008). It is very important to mention that the role of the teacher during such kind of instruction must be thought out in details.

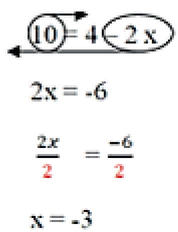
7. Research results

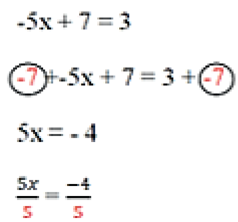
The extended version of research results analysis can be found in (Solarz, 2017). The first part of collected data, I presented in the form of procedures and errors analysis. The procedures were connected with: moving the terms from side to side, using balance method – doing operations on both sides, understanding the structure of expressions, using the equal sign and interpretation of the solution of the equation.

Every solution was annotated with students' remarks that gave justification of performed operations, so I could specify and group the errors. In my doctoral thesis I showed 13 types of lapses connected with 5 categories of procedures. I illustrated them with about 40 examples of students errors, like for example:

$4x = 6x - 2$ (I subtract four from both sides)	$x - 6 = 3x - 6$ (I subtract x from both sides)
$x = 2x - 2$	$6 = 2x - 6$

Among all students' answers, I distinguished those, that showed the influence of the used artefact. Specific writings confirmed, that students' learning was influenced by the experimental tool. Students drew arrows, matched parts of expressions by loops, added terms outside the right and the left side of equations:





The second part of obtained results consisted of students' written responses to the test. The data enabled me to compare the experimental group with the control group. There were different kind of differences between the groups. In such skills like: **performing operations on both sides of equation**, **difficulties with the sign of equality**, **students' own incorrect procedures**, and **no attempt to try** – experimental group made less errors than control group. In other aspects, like **arithmetic mistakes or algebraic transformations**– the control group was better.

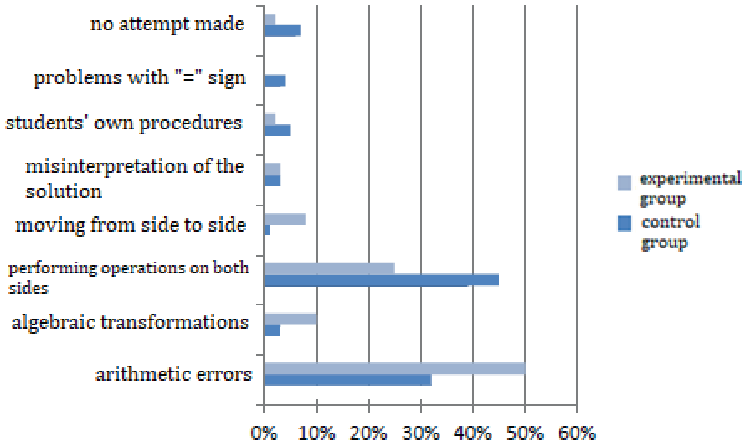


Figure 6: Comparison of the percentage share of mistakes in experimental and control group

The most evident difference you can see, when experimental group was better is the difference in **doing operations on both sides**. I distinguished specific errors, that were connected with this category:

1. type: $4x - x = 4$
2. The operation on both sides is made erroneously
3. Incomplete solution
4. The wrong operation is selected
5. Misunderstanding of expression's structure
6. The operation is made on one side only

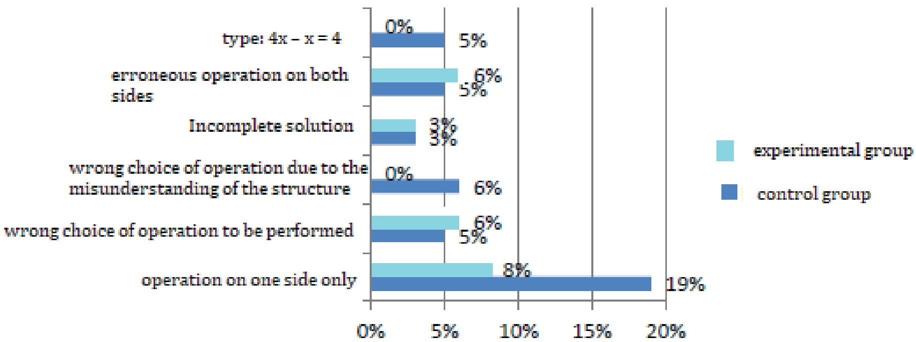


Figure 7: Specific errors

There were less mistakes in experimental group in four cases: 1, 4, 6. In other cases the results were similar. It is also important to mention, that students from experimental group solved equations in various ways – not only using the "scale metaphor", but also doing opposite operations, or moving the terms from side to side.

Summary

To sum up students' work over the course of the experiment and the results of written test, I can highlight **noticeable advantages of using DB**:

- from the beginning students get used to the difference between the "blinking box" and other cards, so they do not confuse the unknown with numbers later on,
- it is impossible to make any move if you do not perform the operation on both sides of the board – the main rule in balance metaphor is also the main one in the game,
- children solve the same problem many times, looking for the best solutions, so they get to know algebraic structure of expressions, considering the order of moves,
- the box can be situated on each side – it does not matter which one.

Finally I have to mention that the DB can give young students oportunity to discover algebraic symbols and the rules of algebraic callculation in specific, uncouncious way, but – obviously – it can not be treated like the only tool in teachers' hands. It is important to look for different kind of artefacts to be able to face up to all students' difficulties in learning algebra.

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