Summary of the thesis

The present PhD thesis is devoted to compute the values of Waldschmidt constants for some configurations of points determined by complex reflection groups. The formal definition of the Waldschmidt constant of a homogeneous ideal in a ring of polynomials is

$$\hat{\alpha}(\mathcal{I}) = \inf \frac{\alpha(\mathcal{I}^{(m)})}{m},$$

where $\mathcal{I}^{(m)}$ is the m-th symbolic power of $\mathcal{I}$.

The main contributions of the thesis can be summed up as follows: The main results of the thesis are as follows:

- We compute the values of Waldschmidt constants for configurations of points determined by some complex reflection groups. We focus on H_3, D_4, B_4, F_4 and H_4 root systems. We provide theoretical proofs on Waldschmidt constants for the above-mentioned configurations with the exception of H_4.

- Moreover, we provide code in the computer algebra system Singular, which estimates the Waldschmidt constant for the above-mentioned configurations and can be easily adopted to other sets of points.

- We compute the resurgence for Z_{20} configuration which contains all 12 points from the D_4 root system and additional 8 points from the F_4 root system.

- Somewhat independently, we prove that the set of points of the H_4 configuration has the geproci property, it is not a half-grid and of course not grid. We explain geproci property below.

Definition of geproci sets of points. *We say that a finite set $Z \subset \mathbb{P}^3$ has a geproci property (the acronim comes from: GEneral PROjection is a Complete Intersection), if its projection from a general point in $\mathbb{P}^3$ to $\mathbb{P}^2$ is a complete intersection.*

This dissertation consists of five chapters. In the first chapter we present notations, basic definitions, theorems and examples from algebraic geometry and commutative algebra, which are necessary to further consideration and for a full understanding of this dissertation.

In chapter two we recall some facts about arrangements associated to finite reflection groups. In this part we consider properties of the D_4 configuration from the point of view of its Dynkin diagram.

The third part of this thesis consists of describing the asymptotic invariants: Waldschmidt constants and resurgences.

The Waldschmidt constants of symmetric sets of points in projective spaces are the core of chapter four.

In the fifth chapter, at the end of this PhD thesis, we present properties of general projections of symmetric sets of points in projectives spaces on the example H_4 configuration. The set of points from the mentioned configuration has the geproci property, it is not a half-grid and of course not grid.

In the Appendix we present some Singular – symbolic algebra program code used to estimate the Waldschmidt constant of aforementioned configurations.