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Mathematics Education in Hungary Today

In Hungary, as in many (if not most) countries, mathematics education recently underwent considerable changes. This process started influencing mass education in our country quite late and is still far from coming to an end. Even so, it may be instructive to take stock of this state of transition, without concealing the difficulties we are encountering.

Let me go back to the origins of our reform. Though the implementation of a new curriculum started only in the seventies, it was based on a pilot work done much earlier:

from 1957 to 1962 as an individual effort, first in grades five through eight in a single school, then in grade two in another school;

from 1963 onwards more systematically, in an increasing number of schools.

During the nineteen-fifties and early sixties I was charged at the Budapest University with courses on mathematics education to prospective teachers of grade 5 through 12. I felt that my words needed factual support; this is why I decided to test my suggestions with an average group of pupils from grade five in five weekly hours.

After three years of our intensive work, Z.P. Dienes arrived on the scene during the summer of 1960. He came from Cracow where he had participated at a meeting of the CIEAEM. In Budapest he delivered a lecture at the Second Hungarian Mathematical Congress (where I, too, reported on my experiences), and conducted some demonstration lessons. What he told and showed us convinced me of the necessity of a new start, one with younger children and a completely different organization.

Learning based on personal experience and small group interaction was appealing to a number of teachers who got acquainted with this approach, but to none of them so much as was needed for the actual undertaking of a full-scale experiment which required an official authorization. For the lack of an abler person, I started the work with thirty-odd second graders and continued it during the 1961/62 academic year.

The teacher charged with all other subjects had to be present at my mathematics lessons, to much of her dismay as the children were often leaving their places to get new assignment cards. They were discussing the tasks in each group. As a consequence, the level of noise surpassed her level of tolerance, and my attempts to moderate it were not entirely successful.

At the end of the school year the authorization for me to continue the pilot teaching was withdrawn. The classroom teacher felt relieved. Frankly, so did I.

Then came August 1962. The Bolyai János Mathematical Society organized an International Symposium on Mathematics Education. Among those present one of the key persons was Mrs. A.Z. Krygowska. During the two weeks of this Symposium my views of mathematics learning probably broadened more than during years or even decades before. I have to admit that in some ways I became over-enthusiastic and prone to biassed ideas. If this happened, the last person to be blamed was Mrs. Krygowska - our princess, as the late Willy Servais named her. It was, in fact, *in spite of* her presence and moderating influence that some of us tended to detach ourselves from the firm ground of reality, that of the schools and of the society.

One of the tangible outcomes of the Symposium was that the Ministry of Education - which had withdrawn the authorization to the earlier pilot teaching - invited me to start a new one, on the same lines, except that the actual teaching in the classroom had to be done by regular lower-grade teachers.

One full year was allowed to prepare the curriculum for grades one through four, both the content and the suggested approach. This time two teachers of a school volunteered, and then in September 1963 they started with two grade one classes.

Initially I was present in the classroom much of the time, prepared work-sheets and assignment cards, ond occasionally took over the teachers′ duty. At some of my suggestions the teachers expressed doubts and were often right. In course of time they modified many of their initial ideas, and so did I.

Later I moved from the university for the National Institute of Education. There I found colleagues sympathizing with my work and joining it, too. It was just in time, since my duties accrued. Other teachers of the same school, teachers of other schools - later also from other cities - visited the two experimental classes, and some of them asked for permission to start similar work. They, too, had to be furnished with equipment, and to be visited from time to time.

As the number of pilot classes increased, so did the number of those visiting the classes, and so did the number of those who volunteered to join the scheme. This sort of feedback - where the increase is roughly proportional to the current number - generates exponential growth, and this is what actually happened. The time being quantified in years, the approximate number of pilot classes was found to form a geometric progression. In ten years the number of classes grew from two to two hundred, the quotient of the progression being

$$\sqrt[10]{\frac{200}{2}}, \text{ i.e. about } 1.58$$

Boundary conditions - such as saturation - tend to slow down this kind of growth after a while: the curve flattens down to become a so called logistic curve. This was not noticeable during the first ten years, the two hundred classrooms amounting to only 0.5 % of those in the Hungarian eight-grade schools. (I should mention that the initial scope of the first four grades was subsequently extended to eight grades, i.e. up to the end of the eight-grade "general schools").With this flattening in mind and ignoring other possible effects, we could expect every - or nearly every - class in the Hungarian eight grade schools to get involved some time early in the next century. I for one could not imagine anything more promising than the idea of this kind of organic growth, whether or not I would be alive by that time. Attempts to speed up the natural growth hardly lead to satisfactory results.

Yet this is what happened, unfortunately. High authorities decided that a new curriculum should be developed and implemented for every school subject of the general school. Our project was found worthy enough to serve as the basis of a new mathematics curriculum for the whole country. We, who were involved in the project, sensed the danger, but could not argue strongly enough. After all, those responsible for other curricula were less prepared. "Let other schools - every school in Hungary - enjoy the fruits of your work" - the enticement sounded.

As a compromise, we moulded a milder version of the curriculum adopted in our pilot schools a n d attained that it be implemented gradually:

first in 5 % of the first grade (the number actually became 8 %),

next year 15 % of the new first grade classes while the pathbreakers came to the second grade,

and so on, with 50, 80 and 100 % of the first grade in subsequent years.

In this school-year (in 1984/85) 100 % of the seventh and about 80 % of the eighth graders "enjoy the fruits of our work" - or do they? You may guess that not all of them do.

The gradual implementation was intended to simulate the organic growth, but it did not work well, by various reasons. First, because the pace was too quick. Second, because it was usually not teachers to volunteer to start with the new curriculum, but their heads or higher authorities. Many of those teachers who had been suspicious of our pilot work when it had no official recognition, hastily jumped on the band-wagon as soon as they found it had one. (This made the originally planned five percent to become eight.) Thus even in the early stages of the implementation, and then increasingly, many teachers were o b l i g e d to follow the new curriculum even though they would have preferred to keep the old one that they felt at ease with. And so the good reputation of our project turned to its fate. We became compulsory, as a state religion or a state ideology. The landlords - the educational authorities - converted the masses, and the latter had to follow the new faith: a practice bequeathed from the late Middle Ages. "Cuius regio, eius religio" - to whom belongs the region, belongs the religion.

It often happened that when an inspector appeared, the teachers came out with some kind of activity to indicate that they were true followers. In other cases, the teacher was happy with the new curriculum, but the inspector or the head teacher was not, and tried to discourage the teacher. The "Cuius regio, eius religio" principle worked in two ways. It still does.

The teacher training - both the initial and the in-service training - should serve to remedy the situation, but their efficiency is not spectacular. Much depends on the trainers, of course. Imparting new knowledge is relatively easy. Difficulties arise when trainees are supposed to unlearn obsolete concepts, to abandon familiar views, to change habitual practices, or - most difficult though most important of all - to change their attitudes. I mean, for instance, accepting children as fellow-learners whose ways of thinking, silly as they seem, merit serious attention - not a standard attitude on the part of Hungarian teachers, I must say. I will come back to this point and illustrate it with an example.

Before doing so I have to outline some features of our curriculum as it emerged in our pilot classes. May I stress the word e m e r g e d? It has been, in fact, the result of a co-operation between the teachers who did the actual daily work and the leaders who were regularly present and often took over the teachers' role.

The content and the structure of the curriculum were influenced by two endeavours: first, to come close to the ways children of the given age think and feel; second, to remain close to the traditional curriculum, unless this contradicts to point one. As to the first, we made efforts not to reduce the content to mere arithmetic. After all, children spontaneously develop other kinds of mathematical ideas, about space, relations, chance etc., even before they go to school. It belongs to the responsibilities of school to nurture the existing germs of such ideas in children's minds. With this in view, instead of asking w h e n to teach geometry or probability we would ask the question: w h a t kind of mental food would serve best in grade one, then in grade two etc. the nurturing of those germs to let them bud and blossom? This is, if you like, an application of Jerome Bruner's famous principle to school situations.

Yet it can, as you know, lead to disastrous results unless it is moderated by common sense and by constant consideration of children's immediate responses as well as of the long-range effects on their development. The traditional curriculum and the teachers' habitual practices may not be adequate to the requirements of our rapidly changing world, still they do have their riches: in course of time they accumulated much of common sense. This again is a question of attitude as well: without our highest respect toward the teaching profession and the individual teacher how could we expect teachers to respect the children?

Instead of describing in details how these two complementary endeavours shaped our i n t e n d e d curriculum, let me give you two examples. They will shed some light (I hope): the first on the i m p l e m e n t e d curriculum, as it is put into practice by teachers, the second on the a t t a i n e d curriculum, i.e., the effect it had on children. Both examples will reflect the difficulties we encounter but favourable auspices will not be totally lacking.

The first example is a story around a problem in our math workbook for third graders. Here is the problem:

> Somebody tells a joke on Monday to five persons. Next day, Tuesday, each of the five tells the joke to six other persons. Each of the latter tells it to seven persons on Wednesday. How many will have heard it on Wednesday?

In an interview published in one of our national newspapers a mathematician complained of this problem, mentioning the dilemma of his nephew. The boy had it as homework, and found three different solutions, depending on the interpretation of the text:

a) Five persons heard the joke on Monday, five times six or 30 on Tuesday, five times six times seven or 210 on Wednesday. The answer is 210.

b) In another interpretation the answer is 5+30+120 or 245. Those who heard the joke on Monday or on Tuesday w i l l h a v e h e a r d i t on Wednesday together with the 210 who heard it precisely that day.

c) He who told the joke to the first five persons must have heard it previously - unless he invented it - so the answer is 246.

The nephew was desperate. "If I come up with any of these solutions" - he said to his uncle - "the teacher may have in mind another solution, and she will make a fool of me before the class because I could not find the *real* solution. The whole class will laugh at me!"

The point of the story is the inference drawn by the mathematician. He said: "Problems in math workbooks should be more cerefully worded so as to exclude different interpretations".

I wonder what you think about it, but I do like giving children problems which they can interpret in different ways. Finding different interpretations is a first step toward inventing problems on their own, or toward mathematizing an open situation. Such activities are at least as important as solving ready-made problems, *if learning mathematics has any goal beyond itself*. In some cases - test items, contest problems - unambiguous wording may be a virtue. You see, there is a watershed here: either you judge that the *main goal of mathematics problem solving is to enable pupils solving further, more difficult problems* in order to pass tests and to win contests, or you see something - may be a lot - that can be achieved *beyond that*. In the first case you will see no point in ambiguous problems or open problem situations - in the second case you will.

In our example the boy was really in trouble, yet actually not because of the ambiguous problem itself, but because it was set by an authoritarian teacher. Let me argue in favour of the interviewed mathematician: as long as there are authoritarian teachers in the schools, problems in the workbooks should be unambiguously worded, or pupils will get in trouble. So let us ban such problems to the detriment of some major goals of mathematics learning?

There is no easy solution to this dilemma. The policy we adopted is: not to ban ambiguous problems, but to convey to teachers how to make use of them. Authoritarian personalities will not change overnight but are they really what we suppose them to be? May be the teacher of the boy in our story only needs an encouragement, and she will accept different answers to our problem, depending on the children's interpretations.

She may even find it fun to suggest them further interpretations. 246 seems to be the greatest number. But is 210 the smallest? What does Fig.1 tell you? Yes, on Tuesday

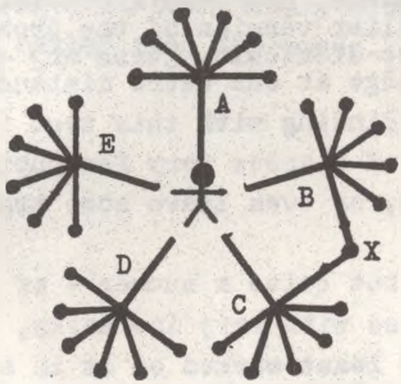

Fig.1

somebody (X) hears the joke from two different persons (B and C) out of the five who heard it the day before. He is polite and does not disclose the second time to have heard it already. So Monday 5, Tuesday 29 - and on Wednesday? 29 times 7. Could it be less? Still less? Children compete in bringing the number down and further down. They use their imagination - the right hemisphere of their brain, if you like - and they use the other hemisphere, too, in producing and defending their answers. Whether they come down to the number seven - each of them listening to the joke Wednesday as many times as there are persons who heard it Tuesday - is almost irrelevant. The main interest is not in the minimum solution, but in a succession of good, better and still better solutions. The situation will probably make children laugh. (By the way, not at the expense of a mate of theirs).

Let me turn to my second example, the one throwing light on the attained curriculum. Fifth graders struggled with the following problem - just one item in a battery - some time in May (age at the beginning of the school year: 10^+).

> The area of a rectangular flower bed is 36 m^2. Right on the edge of the flower bed there is a rope surrounding the flower bed, with knots at one meter distances everywhere. How many knots are there on the rope? (In an earlier version of the problem - which proved to be more difficult, guess why - tulips were planted on the edge at one metre distances).

The interesting finding with this test item was this:

in the majority of classes very few pupils or nobody at all could solve it, or even leave some trace of an attempted solution;

in a minority - but quite a number - of classes most pupils, including those with very low marks, found one or more solutions, or at least worked on it in a way which made sense, even if they mis-calculated the answer.

The social background of the pupils in the first and in the second type of classes showed no difference that could explain the finding. Further inquiry revealed one single reason: the "good" classes were "good" because their teachers got pupils used to experiment with concrete materials such as coloured rods, or geoboards, or just making drawings on their own in order to figure out solutions to problems. The classes with low average scores on this item were sometimes quite good in routine problems and calculations. At this item they were blocked probably because the problem was not in stock in their heads. They could not recall having ever met any similar problem, so they did not do anything.

The picture furnished by these examples and comments is necessarily insufficient and may be biassed, too. I did not mention all the mistakes made during the reform, including some quite serious blunders. We are working just now on eliminating their effects. Nor did I speak of the educational - didactical or other - problems which are still open, and cannot be settled without further reasearch and deeper insight into the learning process. But we cannot wait until they are. Time urges us to decisions about approaches suggested by the programmes, textbooks etc., even if some of these decisions may later prove to be wrong.

Let my last word still be an expression of hope that the recent changes in mathematics education in Hungary have been more for better than for worse.