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Learning system based on cellular automata

Abstract. In this paper we present the CELLS2 system which is a learning system based on cellular automata. In the learning process CELLS2 used the Quine-McCluskey method which minimalizes boolean formulas.

1. Introduction

One of the most interesting assignments inspiring researchers of artificial intelligence is programming digital computers so as they would have a learning property. Structures which have a knowledge discovery ability from the training information are called learning systems. The goal of this article is formal introduction into learning system structure CELLS, which way of working is directly connected with the cellular automatas evolution rules.

2. CELLS system

Let the nonempty set C, natural number m and m-tuple ordered structure $L \in C^m$ be given, where C^m is the Cartesian product of m copies of the set C, which means that

$$L = \langle c_0, c_1, c_2, \dots, c_{m-1} \rangle, \quad \text{with} \quad c_i \in C, \quad i \in \{0, 1, \dots, m-1\}.$$

The n-tuple ($n > 1$) subset $\{L_1, L_2, \dots, L_n\}$ of the C^m set will be denoted by C_A. Evidently

$$C_A = \bigcup_{1 \leqslant i < n} L_i,$$

where

$$L_i = \langle c_{i_0}, c_{i_1}, c_{i_2}, \dots, c_{i_{m-1}} \rangle, \quad i \in \{1, 2, \dots, n\}.$$

It is assumed that there is a function indexing the elements of the set C_A.

Let L_{i_p} mean periodical displacement of L_i with p steps to left

$$L_{i_p} = \langle c_{i_p}, c_{i_{p+1}}, \dots, c_{i_{m-1}}, c_{i_0}, \dots, c_{i_{p-1}} \rangle,$$

where $0 \leqslant p < m$. Let the ordered pair be given

$$F_{i_p} = \langle L_{i_p}, c_{(i+1)_p} \rangle.$$

The set of all these pairs will be marked by F^*:

$$F^* = \bigcup_{\substack{1 \leqslant i < n \\ 0 \leqslant p < m}} F_{i_p}.$$

It should be noticed that F^* is a relation in the Cartesian product of C^m and C:

$$F^* \subset C^m \times C.$$

DEFINITION 1

The set C^m will be called the one-dimensional cellular automata if and only if the set F^* is a function, which means that the following property holds:

$$\bigwedge_{\substack{1 \leqslant i < n \\ 1 \leqslant i^* < n}} \bigwedge_{\substack{0 \leqslant p < m \\ 0 \leqslant p^* < m}} L_{i_p} = L_{i^*{}_{p^*}} \rightarrow c_{(i+1)_p} = c_{(i^*+1)_{p^*}}.$$

Let such a subset F^{2*} of the set F^* be given, that the elements F_{i_p} with odd index i belong to it, which means

$$F^{2*} = \bigcup_{\substack{1 \leqslant i [i \text{ odd}] < n \\ 0 \leqslant p < m}} F_{i_p},$$

where n is even.

DEFINITION 2

Such a set C_A that the family F^{2*} of the pairs F_{i_p} created on it makes up a function will be called incomplete one-dimensional cellular automata.

Evidently every one-dimensional cellular automata is also an incomplete one-dimensional cellular automata. Only incomplete one-dimensional cellular automatas will be the subject of the following discussion.

Let the set $Z = \{0, 1\}$ be given and let h_Z mean the m-tuple ordered structure $h_Z \in Z^m$ called the history h_Z.

DEFINITION 3

The reduction L_i with respect to h_Z will be called a function mapping an m-tuple ordered structure L_i into an m'-tuple ($m' \leqslant m$) ordered structure $L_{i|h_Z}$. Only those elements from L_i for which elements from h_Z with the same index have a value 1 will belong to $L_{i|h_Z}$.

The ordered structure L_{i_p} reduced with respect to the history h_Z will be denoted by $L_{i_p|h_Z}$. The pair F_{i_p} with such a first element will be marked by $F_{i_p|h_Z}$. The set F^{2*} composed from the above pairs will be denoted by $F^{2*|h_Z}$. It follows from the given definitions that $F^{2*|h_Z} = F^{2*}$ if and only if the history h_Z consists of only ones.

DEFINITION 4

A number of ones in the history h_Z reducing elements L_{i_p} is called the index $|F^{2*|h_Z}|$ of the set $F^{2*|h_Z}$.

A real number μ_F form the interval $\langle 0.1\rangle$ can be attributed to every element $F_{i_p|h_Z} \in F^{2*|h_Z}$ in the following way:
Let s be a number of pairs of the set $F^{2*|h_Z}$ with element L_{i_p} identical to the first element of the analyzed pair, and let l be a number of pairs in the number s which element $c_{(i+1)_p}$ is identical to the second element of the analyzed pair. Then:

$$\mu_F = \frac{l}{s}.$$

The function μ attributing the value μ_F to every element $F_{i_p|h_Z} \in F^{2*|h_Z}$ will be called a characteristic function of the set. The set $F^{2*|h_Z}$ with the function μ will be identified with the fuzzy set.

DEFINITION 5

The following set denoted by $F^{2*|h_Z|\alpha}$ will be called the α-section of the set $F^{2*|h_Z}$:

$$F^{2*|h_Z|\alpha} = \{F_{i_p} \in F^{2*|h_Z} : \mu(F_{i_p}) \geqslant \alpha\}.$$

THEOREM 1

For every set $F^{2*|h_Z}$ there exists such an α-section $F^{2*|h_Z|\alpha}$ that is a function.

Proof. To prove this theorem it is sufficient to consider any α-section with $\alpha > 0.5$. As the function μ attributes a real number from the interval $\langle 0, 1\rangle$ to elements of the set $F^{2*|h_Z}$, the sum of values of μ_F for pairs with identical elements L_{i_p} equals 1. Therefore, if there exists an element $F_{i_p|h_Z}$ for which $\mu_F > 0.5$, then there exists at at most one element of the set $F^{2*|h_Z|\alpha}$ from the pairs with identical first element fo which $\mu_F > 0.5$. The α-section with $\alpha > 0.5$ separates at most one element from each pair with identical first element L_{i_p}. That is the suffucient condition for the set $F^{2*|h_Z|\alpha}$ to be a function.

Let the set F^{2*} be a function.

DEFINITION 6

A process of finding a set $F^{2*|h_Z}$ with the smallest index being a function is called the minimization of the set F^{2*}.

To minimize a set which is not a function one should appeal to the Theorem 1.

DEFINITION 7

A process of finding a set $F^{2*|h_Z|\alpha}$ with the smallest index and the smallest value of α being a function is called the α-minimization of the set F^{2*}.

The problem of the α-minimization of function F^{2*} is searching the two-dimensional space of possible values of α and h_Z. The solution to this problem is an ordered pair $\langle \alpha, h_Z \rangle$ called the optimal pair. For specific sets F^{2*} many different optimal pairs can exist.

Now a few terms from a field of artificial intelligence will be introduced. Let K be a nonempty set of ordered pairs. The set K will be called a knowledge.

DEFINITION 8

Any nonempty subset of the set K is called training information. Ordered sets with elements being vectors called an input vector (a question) and an output the vector (an answer) form training information.

DEFINITION 9

A structure which after analyzing a finite set of training information is able to generate an algorithm of obtaining the correct answers to the questions from the set K is called a learning system with supervision.

After introducing the above definitions it is possible to define the learning system CELLS. CELLS is a type of the learning system with the supervision. The main step in the construction of the CELLS system is identifying the training information in it with the set F^{2*}.

DEFINITION 10

A system able to perform α-minimization on the training information set F^{2*} is called the learning system CELLS.

3. Conclusion

A computer program based on the given formalism shows large ability to learn, thus proving usefulness of creating the learning systems based on the cellular automata.

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