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On a test concerning the notion of the set

Abstract. Many students have problems with solving exercises concerning sets which elements are also sets. Many wrong solutions are due to incorrect understanding of the term set. We suggest a test which is aimed at drawing attention to the above problems.

In this paper we consider a test which is related to problems concerning the notion of the set. These problems are connected with the double understanding of this notion. Especially it is conspicuous in exercises in which there are families of sets (see [1], [2]). In addition to correct understanding of the notion of the set (i.e. in the distributive sense), there is also an incorrect one (in the mereological, collective sense). In the second case elements of sets are treated as pieces, parts, ingredients or fragments. Let us consider the expression "A is an element of the set of b's". In the distributive sense it means simply that A is a b (see [3]). If, however, we interpret the terms "element of" and "the set of" collectively, then our original expression means that A is a (proper or improper) part of the object that has the following properties:

1) every b is a part of it,
2) every part of it has a common part with a b.

In Leśniewski's mereology (see [4]) the relation of "being an element of" is reflexive, transitive and antisymmetric, thus it is a partial order. The relation of inclusion in the set theory has the same properties; however, the relation "belonging to a set" has not such properties. Generally, many pupils (and students) do not distinguish these relations. The relation "being an element of" ($a \in A$), inclusion ($A \subseteq B$) and "being a part of" ($A \leqslant B$) are identified by them. This situation is due to many reasons. One of them is an effect of replacing mathematical notions with notions taken from informal speech. The following test may be very useful in verifying the pupils' (students') mistakes and may contribute to establishing the notion of the set in the distributive sense.

Test checking intuitions concerning the notion of the set

1. Let A and B be sets. It is true that:

a)	$A \subset \{A, B\}$	YES □	NO □
b)	$B \in \{A, B\}$	YES □	NO □
c)	$\{A\} \subset \{A, B\}$	YES □	NO □
d)	$\{B\} \in \{A, B\}$	YES □	NO □

2. It is true that:

a)	$1 \in \{\{1,2\},\{3\}\}$	YES □	NO □
b)	$\{4\} \subset \{1,\{2,3\},4\}$	YES □	NO □
c)	$\{2,3\} \subset \{1,\{2,3\},4\}$	YES □	NO □
d)	$\{2,3\} \in \{1,\{2,3\},4\}$	YES □	NO □

3. Let $Z = \{A, B\}$, where A is the set of all even numbers and B is the set of all odd numbers, $\emptyset$ is the empty set. Answer the following questions:

a)	$2 \in A$	YES □	NO □
b)	$2 \in Z$	YES □	NO □
c)	$\{2\} \subset Z$	YES □	NO □
d)	$\{2\} \in A$	YES □	NO □
e)	$\emptyset \in Z$	YES □	NO □
f)	$\emptyset \subset Z$	YES □	NO □
g)	How many elements does the set Z have?	...	
h)	How many elements does the set A have?		

4. Let A, B and C be sets; $\emptyset$ is the empty set. The intersection of the sets $\{\emptyset, A, B\}$ and $\{\emptyset, B, C\}$ has the form:

a)	$\emptyset$	YES □	NO □
b)	B	YES □	NO □
c)	$\{B\}$	YES □	NO □
d)	$\{\emptyset\}$	YES □	NO □
e)	$\{\emptyset, B\}$	YES □	NO □
f)	$\emptyset \cup B$	YES □	NO □
g)	$\{\emptyset\} \cup \{B\}$	YES □	NO □

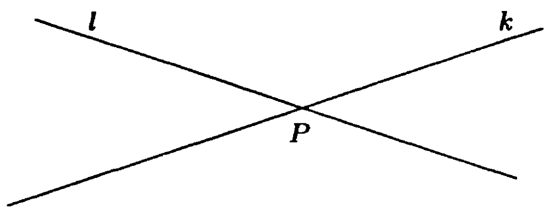
5. We consider the complement of the set of all nonnegative numbers in the set of all natural numbers more than 5. What will we obtain?

a)	such operation is impossible	YES □	NO □
b)	$\emptyset$	YES □	NO □
c)	$\{1,2,3,4,5\}$	YES □	NO □
d)	$\{-1,-2,-3,-4,-5,0\}$	YES □	NO □

6. Which of the following sets is the set of solutions of the equation: $x^2 = 1$?

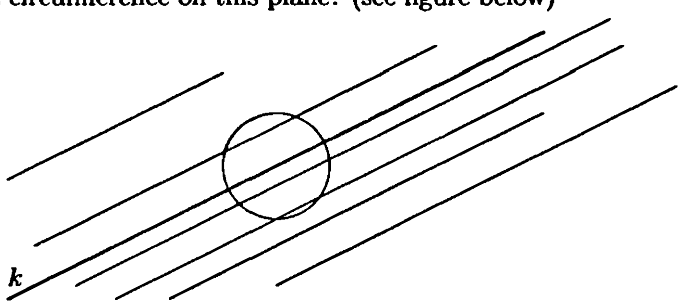
a)	$\{1\}$	YES □	NO □
b)	$\{1,-1\}$	YES □	NO □
c)	$\{1\}\cap\{-1\}$	YES □	NO □
d)	$\{-1\}$	YES □	NO □

7. Lines k and l are not parallel and they are included on the same plane. They have a common point marked by P (see figure below). Which of the following notations are correct?



a)	$k\cap l = P$	YES □	NO □
b)	$k\cap l = \{P\}$	YES □	NO □
c)	$k\cup l = \{P\}$	YES □	NO □
d)	$k\cup l = P$	YES □	NO □

8. By the direction determined by a line k (on a plane) we understand the set of all lines parallel to k. What is a common part of this direction and a circumference on this plane? (see figure below)

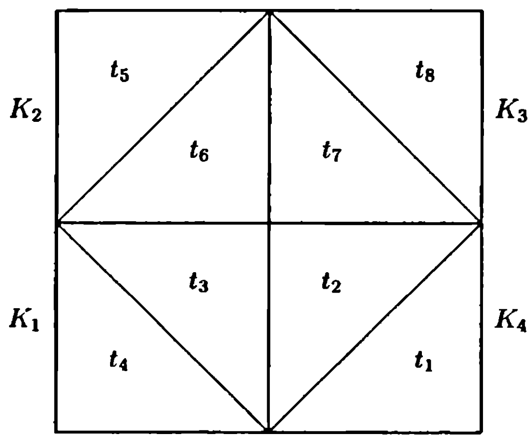


a)	the circumference	YES □	NO □
b)	whole plane	YES □	NO □
c)	$\emptyset$	YES □	NO □
d)	a common part does not exist	YES □	NO □

9. Let A be the set of five bottles filled with a liquid. The liquid was poured out. Now the set A consists of:

a)	five empty bottles	YES □	NO □
b)	five bottles filled with liquid	YES □	NO □
c)	$\emptyset$	YES □	NO □
d)	five empty bottles and the liquid which has been poured out	YES □	NO □

10. We consider two sets: A – the set of the squares K_1, K_2, K_3, K_4 that is $A = \{K_1, K_2, K_3, K_4\}$ and B – the set of the triangles t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8 that is $B = \{t_1, t_2, \ldots, t_8\}$ (see figure below)



Which of the following relationship are true?

a)	$A = B$	YES □	NO □
b)	$A \subset B$	YES □	NO □
c)	$B \subset A$	YES □	NO □
d)	$A \cap B = \emptyset$	YES □	NO □
e)	$A \cup B = B$	YES □	NO □
f)	$A \cup B = A$	YES □	NO □

11. Let $A = \{M, D\}$, where M is the set of all boys in a school and D is the set of all girls in a school. Does the set A consist of all pupils in the school?

YES □ NO □

12. Let $[AB]$ be a sector with the ends A and B, $A \neq B$. The set of all its interior points is removed. What will we obtain?

a)	two points A and B	YES □	NO □
b)	$\{A, B\}$	YES □	NO □
c)	$\emptyset$	YES □	NO □
d)	such a set does not exist	YES □	NO □

13. Which of the following statements are true?

a)	every line is the set of points	YES □	NO □
b)	every line is the set of sectors included on this line	YES □	NO □
c)	every plane is the set of points	YES □	NO □
d)	every plane is the set of all lines included on this plane	YES □	NO □

14. Which of the following relationships are true?

a)	$\emptyset \in \{\emptyset\}$	YES □	NO □
b)	$\emptyset \subset \{\emptyset\}$	YES □	NO □
c)	$\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$	YES □	NO □
d)	$\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$	YES □	NO □
e)	$\emptyset \in \{\emptyset, \{\emptyset\}\}$	YES □	NO □
f)	$\emptyset \subset \{\emptyset, \{\emptyset\}\}$	YES □	NO □

15. In which of the following cases:

a)	$A = \{a, b\}$ and $B = \{a, \{a, b\}\}$	YES □	NO □
b)	$A = \{a, b\}$ and $B = \{\{a, b\}, c\}$	YES □	NO □
c)	$A = \{a, b\}$ and $B = \{\{a\}, \{b, c\}\}$	YES □	NO □

the sentence:

If $a \in A$ and $A \in B$, then $a \in B$

is true?

References

[1] G. Bryll, R. Sochacki, *O dwóch różnych pojęciach zbioru*, Dydaktyka Matematyki **19** (1997), 5–13.

[2] Z. Dybiec, *O pewnej trudności związanej ze specyfikacją w nauczaniu matematyki*, Dydaktyka Matematyki **8** (1987), 213–220.

[3] K. Kuratowski, A. Mostowski, *Teoria Mnogości*, Monografie Matematyczne t. 27, PWN, Warszawa 1966.

[4] St. Leśniewski, *Collected Works, v. I, II*, PWN–Kluwer Academic Publishers, Warszawa 1992.

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