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Pavol Hanzel

The finite subsets of natural numbers

Abstract. The article discusses relationships between triangle and square numbers. The stress is laid upon numerical equality. A finite set of these numbers and its subsets are given by software. Finding natural numbers with a specific property is of great didactic value. This topic was included into the course of study preparing future elementary school teachers. The article deals with comparison of mathematical problem solving procedures by 1^{st} and 4^{th} year elementary school student teachers. The method of experiment was applied. Accessible software is used in solving more complex problems in the 4^{th} year.

Figural Numbers

The preparation of elementary school teachers of mathematics in arithmetics, which is based on the idea of natural numbers and calculations in the decimal numeration systems, plays an important role.

The students of the School of Pedagogy proceed through different levels of scientific knowledge during the time of preparation for their future career. In the first school year students encounter certain difficulties with analyzing basic arithmetical terms and operations. An interpretation of terms, which helps the students understand the nature of natural numbers (as cardinal numbers) is done through finite subsets of "small numbers". As a typical example we present tasks which consist in comparing triangular and quadratic numbers.

P1. From 15 circlets we can organize a triangular number which is not possible to be organized as a quadratic number. See the below picture.

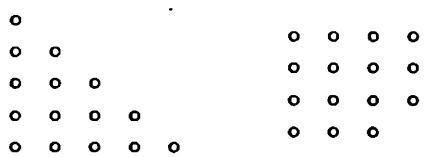

Figure 1.

Find such a triangular number that can be transformed to a quadratic number.

The solution is very easy, not only for the first year elementary school teacher students studying mathematics, but also for people who are not involved in mathematical aspects of the problem. Solution of the task is the following: "the bottom row of the triangular number has to have eight circlets." To find an alternative solution is not such a trivial task even for the first year student with sufficient knowledge about the terms and nature of natural numbers. Therefore finding a solution to task P1 involves using the method of experiment. Gradually and alternately we complement the circlets:

1. as the first step we added one circlet to the pattern on the right so that we get a quadratic number (it will have 16 circlets),
2. then we add 6 circlets to the triangular number (we will get 21 circlets),
3. then we add 9 circlets to the quadratic number (we get 25 circlets),
4. repeat steps 2 and 3 until we obtain 36 circlets,
5. the solution is quadratic number $S_{6,6}$ (the size of the side of the square is 6 circlets) or triangular number T_8 (8 circlets in the bottom row).

The Method of Experiment

The use of the method of experiment

"1st experiment — 2nd experiment — ... k. experiment = result"

is on one hand illustrative and on the other hand motivational. During our work, we found a solution of our task in a relatively short time. Further, we found an algorithm suitable for application in microcomputers. The reader may be acquainted with the method,

"experiment — hypothesis — verification"

in the work of prof. J. Kopka [3].

Using similar analogical tasks, we require the student to solve a problem using the experimental method and find at least one solution that is appropriate for first year students. Students enjoy solving such problems and they overcome their fear of mathematics. Mathematics becomes interesting and they lose the aversion they brought from the high (secondary) school.

It needs to be mentioned that P1 can be modified in a variety of ways. For example:

- How many circles will there be in a triangular number if the bottom row is to contain 10, 20, 50 numbers?
- Is there a triangular number that can be doubled to also obtain a quadratic number?
- Find at least two solutions to P1!

All modifications of this task require work with small number subsets of natural numbers.

The method of increasing the level of abstraction, but also complexity of final subsets, leads the student to a higher level of complexity where knowledge and skills can be strengthened by creativity. The highest level of skill is achieved in the final year of university studies, where there is an opportunity to work with computers and learn the basics of appropriate software (Excel 2000 is sufficient for finding the solution to problems like P1). Later on the student is also capable of finding pairs of solutions to P1. For example, solutions to P1 include 1 225, 41 616, 1 413 721, 48 024 900 circlets.

In the spirit of the terminology of "figure numbers" invented by Pythagorians there exist quadrant numbers

$$S_6, \quad S_{35}, \quad S_{204}, \quad S_{1189}, \quad S_{6930}, \quad \ldots$$

or triangular numbers

$$T_8, \quad T_{49}, \quad T_{288}, \quad T_{1681}, \quad T_{9800}, \quad \ldots$$

Using software as an innovative element

The following thoughts will be addressed to students of the final two years, who use PC technology in finding solutions to mathematical tasks as well as searching for suitable information on the internet. Today there exist many high quality web sites that contain numerous mathematical information and provide suitable resources for upper level students. Almost every department of mathematics in Slovakia offers information written by local researchers as well as recommended web links that can be used for individual creative study. In this way, students can receive "innovative elements in their education" as they are described in the work of [2] P. Klenovcan. For example, the department of mathematics offers on its web site links regarding

History of Mathematics
The Mac Tutor History of Mathematics Archive.

Modification of Tasks

Using tasks of end subsets of natural numbers is appropriate in the education of future teachers even in the case when they only have basic knowledge in number theory. Let us introduce a couple of interesting topics:

- Fibonacci numbers and the above mentioned work of prof. J. Kopka, which presents Fibonacci's considerations on the reproduction of rabbits in a very approachable and elegant manner.

- ☐ Sparse subsets. Studying the sparse elements of P1 is interesting. Very easily it can be shown that this subset is sparse.
- ☐ Golden division and constants of golden division.
- ☐ Prime numbers and coding with the use of prime numbers, utilizing the system of falling doors. These tasks create a pleasant and motivating atmosphere. (Safely send a message if you have two different hanging locks!)
- ☐ Solve modified tasks about Fibonacci numbers. Consider that rabbits are not immortal. There exists a certain percentage of losses (sickness, epidemics, etc.), such as imposed loss (e.g. theft) but also losses by natural death. With such limitations, solving the problem becomes progressively harder.

In solving these problems of end subsets with certain limitations we encourage fourth year students to use Excel. This software allows students to create a useful didactic tool.

The difference between first year and fourth year students is only in the type of tool used. In the first year it would be pen and paper, but in the fourth year it would be a computer that becomes "a didactic technical tool without being a study tool" [4]. Students note that the process is equally satisfying both at the beginning and at the end of their studies.

The purpose of this article was to outline possibilities of developing a mathematical culture as described in the work of P. Klenovcan [1]: "During their course of study future teachers of mathematics should among other things systematically cultivate a certain mathematical culture which includes the need to define, to prove, to abstract, to think logically, and creatively."

References

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Department of Mathematics
Pedagogic Faculty
Matej Bel University
Ružová ulica 13
974 11 Banská Bystrica
Slovakia
E-mail: phanzel@pdf.umb.sk