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On children's understanding of area

Abstract. Area and volume belong to the most important geometrical concepts due to their broad application outside mathematics. Certain problems connected with the understanding of "area" observed in 11 year old children will be presented in the paper.

"Area and volume belong to the most important geometrical concepts due to their broad applications outside mathematics. And they are not easy ones" ([7]). Restriction to the concept of the area of a rectangle and a square, focusing on formulae, can create undesirable difficulties at higher levels of education. Therefore, the concept of area should be "introduced in a way as general as possible ", i.e. so that it be applicable to various figures; without excessive exposing formulae; by forming its informal intuitive understanding ([7]).

It is common knowledge that any change to the concepts once formed in a child's mind, a change consisting even in generalization only, is very difficult, and any didactical activities undertaken in this direction prove often to be completely unavailing.

Difficulties concerning the understanding of the area of a figure, in particular the area of a polygon, arises often in classroom practice. While researching into teachers' mathematical knowledge in USA and China ([2]) one of the four tasks given to subjects concerned the perimeter and area of a figure:

Let us assume that the topic of the lesson is the perimeter and area. A certain pupil is satisfied with his "discovery": Area increases along with perimeter. He confirms his sort of "theory" with an example of the rectangle 4×4 *which is changing into a rectangle* 4×8*: the perimeter increases from 16 to 24, and the area from 16 to 32. What would you say to this pupil?*

While answering this question, majority of the interviewed American teachers was able to cite the formulae for the perimeter and area of the rectangle. When it came to analysing their mathematical aspect, the teachers felt confused. In most cases a closed figure was replaced with a rectangle. For Chinese teachers this question also turned out to be a challenge. However, a majority of them, upon deeper reflection, was able to come up with adequate counterexamples.

The same question was presented to math freshmen of the Rzeszow University ([3]). Many students were not able to answer this question, some of them stated openly that they did not know, some appeared looking for appropriate examples. The table below shows numbers of correct answers to the four questions given by teachers and college students, in particular the answers to the question 4.

Task	Students	American teachers	China teachers
1.	58%	20%	86%
2.	55%	40%	90%
3.	26%	4%	90%
4.	**26%**	**4%**	**70%**

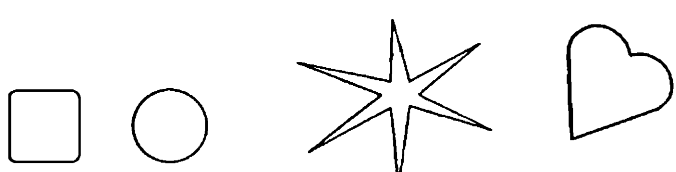
Table 1.

In classroom practice pupils often confuse the concepts of area and perimeter, particularly when it comes to a square or a rectangle. They do not know which of these concepts to use in a specific problem situation. Many a time they use wrong units to express the measurements. These difficulties can be reduced through a deeper treatment of each of these concepts and referring to a realistic context. Both these concepts characterize the size of a figure. However this can be viewed in many different ways and very subjectively. When we ask which of the dogs is bigger (fig. 1a), motivation for an answer can be completely different from stating which of the leaves is bigger (fig. 1b).

a) b)

Figure 1.

Comparing sizes of figures may seem sometimes intuitively easy (although in reality we are not sure what aspects are taken into account by a pupil while comparing the figures) and sometimes even impossible (fig. 2a, 2b).



a) b)

Figure 2.

Area and perimeter are attributing a number to the figure so that one could say which one is bigger when one cannot do it otherwise. Teaching should lead to associating the area and perimeter with a situation in which we want to compare figures and at the same time make the pupil aware that what is bigger in one aspect may be smaller in another one.

As a teacher I faced, not for the first time, the following problem: *How to talk with pupils about area of a figure?*. "It is not easy to come up with such practical problems which could be useful in the introduction of calculating the area of a figure. In order to dress this concept in a conceptual structure, this problem should be referred to other notions already known to the pupil and related to the notion of area" ([1]).

Generally, pupils go through problems concerning the area of a figure at grade 4 (10-11 year pupil) after topics connected with measuring the length of a broken line and the perimeter of a polygon. While teaching how to measure and calculate the perimeter I did my best so that the pupils were able to use different units while measuring. The perimeter of a figure, in school practice reduced to a polygon, was defined as a sum of its sides. Based on this fact children calculated the perimeters of various figures, among others, of a rectangle and a square.

The area of a plane figure is understood as the minimal number of unit squares needed to fill completely the figure by placing them side by side (fig. 3).

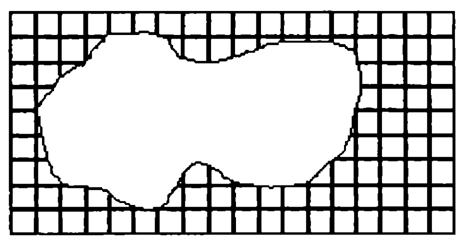


Figure 3.

In an analogous way the volume of a solid is defined. The difference consists in replacing unit squares with unit cubes.

It seemed to me that the area of a figure, regarded as (bounded) part of the plane, in a specific situation as an area (field) to be sown out or a board to be painted, in a natural and obvious way characterize the size of a figure. I began with the question of finding the size of a figure. Pupils were divided into groups of three persons. Each of them was given a sheet containing various figures (fig. 4). They were to answer the following question: *Which of the figures is the biggest?*

However, the drawn figures were not "simple" that I did purposfully. I was aware of the fact that the children would know neither how to compare the size of the drawn figures nor how to assess the size of each one. Therefore I suggested cutting these figures. I also prepared squared foil sheets so that the pupils could cover the figures with them and then count the squares inside of each figure, but I did not want to give them out to the pupils at once. I also anticipated they would have difficulties in counting the squares in "triangles" — I planned discussing this problem over with the children. I thought associating the size of a figure with what "is inside" would be natural for them and only they would lack a tool.

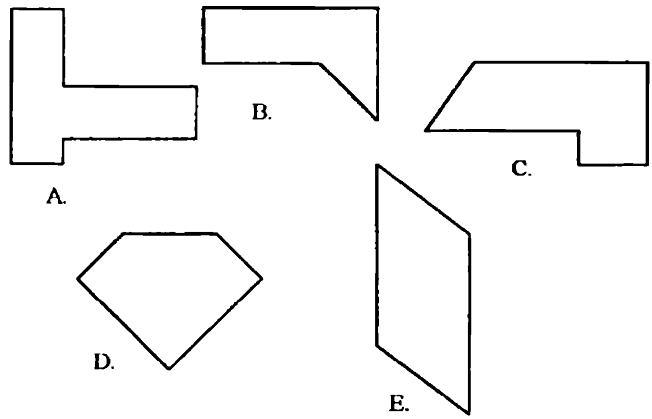


Figure 4.

However, it was not the case. On giving out the sheets with figures on and posing the problem no children seemed to be puzzled about what to do next. They all reached for a ruler and began to measure the perimeters. I think, that in such situations the teacher should give children a chance and wait for their reaction. The fact that they went on automatically to measure the perimeters amazed me. I hoped that after a while somebody would come up with a different strategy. I expressed my interest in their actions and encouraged them to explain their procedure ([4]). The only problem the children noticed was the accuracy of their measurements. They measured the perimeters and arranged the figures in order (of perimeter). I let them finish their work and faced the problem how to make them give up regarding the perimeter as a measure of the size of a figure. As the lesson was finished I asked the children to keep their drawings and calculations for the next lesson.

An analogous situation happened during an experiment described in *Psychological Didactic*, carried out in two schools in Zurich. The research concerned comparing the educational effects of traditional teaching with problem solving teaching and included the area of a figure, in particular, of a rectangle. Two classes took part in the investigation. In the class where activity methods were used (contrary to the one where traditional methods were applied) the following task was posed to the pupils:

From which of the meadows will the farmer get more hay? Model meadows were the rectangles A (2 cm by 4 cm) and B (1 cm by 6 cm).

The interior of the rectangles was not painted. At first, the pupils stated that the area B (meadow) is bigger so the hay crop from this field should be bigger. They measured the perimeters of the rectangles — the perimeter of the rectangle B is 14 cm, and of the rectangle A only 12 cm.

I wondered why in their reaction a certain analogy arose, why they regarded the perimeter as an appropriate measure of the size of a figure. This lead me to suppose that:

- pupils regarded the perimeter as a measure of the size of figures;
- since the drawn figures were not painted inside the pupils treated them as a broken line and calculated its length;
- pupils' reaction was conditioned by their previous experiences connected with measuring the perimeter of a figure and the length of a broken line;
- they responded to my question in the way they understood it so they answered consistently to their thinking;
- perhaps their reaction would be different if the models were figures of the same type (e.g. various rectangles).

I tried then to eliminate some of the sources of such reactions. I prepared a task closer to the specific interpretation of the size: The sheet represents chocolates. They are of the same sort but they have different names. Which of them would you like to have? Why? (fig. 5)

Figure 5.

Generally children like chocolate and probably they would like to have the biggest one. However, in this case they also began to measure perimeters.

Two of the children noticed immediately that all the chocolates have the same perimeter. The chocolates were selected so as to have the same perimeter but different areas. I wrote children's answers on the board: "I want the biggest one!", "They are all the same!" as well as a range of other selections but not based on mathematical aspects.

I asked the children to cut out these chocolates out of squared paper (the pupils could cover the chocolates with squared paper) so that the lines would be arranged straight. Then I asked them to calculated how many children can help themselves to each of the chocolate assuming that each child can take one piece. The children calculated correctly the number of pieces in each figure and stated how many children they could treat each of the chocolate. I returned once again to the question which of the chocolate they would like to have, but still I did not hear the answer I expected. However, upon the way some children behaved, I inferred that they realized that something was wrong; that they were raising some doubts about the role of the perimeter when assessing the size of a figure.

After this experience my attention was brought up to the fact that children might have taken into consideration the fact that the chocolates might have been of different thickness. If it was the case, my question could also have a different meaning — the size of chocolate relates to its volume. However, none of the children's replay implied that different chocolate's thickness occurred to them. The chocolates were represented as plane figures and it is not certain how this affected such interpretation; teamwork or some pupils' swift replies might impose it on the others. The pupils could also follow a previous scheme — the lesson before they measured perimeters. The teacher did not intervene in their actions so they repeated the known schema (they tried to meet the teacher's expectations).

In the next lesson children's task was to investigate the following problem situation: There are different rooms drawn on the paper. Parents decided to floor them with the same tiles. Which room will have the most expensive floor? The children were divided into groups of two persons.

Rectangles, as model rooms, were selected so as two of them would have the same perimeter and different areas; two of them — different perimeter and the same area; one — the least area but the perimeter not the least (so as not to associate little perimeter with little area), another one — the least perimeter and the area not the least (rectangles' dimensions: 6 cm × 8 cm, 3 cm × 1 cm, 4 cm × 12 cm, 3 cm × 13 cm, 6 cm × 7 cm).

The pupils noticed that the rooms are big and flooring would be the most expensive in the biggest room. In one of the groups girls stated: "We calculated with the aid of the squares cut out from the chocolate and got: in Gosia's room 48, Ola's 39, Agata's 42, Jack's 48 and in parent's room 33" but somebody disagreed with them and asked: How do you know these squares are good for doing it? Thus, a problem arose: We do not know how big the tiles are, what shape they are (rectangular or squared). The children themselves, after a discussion, stated: We measure the room not the tiles, we can do it with the aid of rectangles or another ones (*squares*). We can do it differently. It is not important which ones we use but it is important we use the same measure: either all is done in centimetres or in decimetres or meters. It is difficult to obtain the same measurement if we do not know what like the tiles are. Regardless of the fact how big the tiles are, we can always say which room is the biggest depending on the number of tiles used. It can be done in square units, square meters for example — which we use mostly for measuring rooms.

The fragments of the discussion reveal the core of the problem. This time children alone, without teacher's intervention, explained each other a lot, much more then the teacher expected. The first and very important problem is the lack of information about the tiles. How to measure, if we do not know what to use. We can apply different measures but it is important to use the same measure while comparing the figures. Certain pupils could not comprehend this dependence but others tried and explained it to them. The children used a term "to measure in centimetres, in decimetres" but it was evident that they did not mean the length but the tiles — squares or rectangles. They did not know how to express it. This problem was eventually solved by a pupil stating that a square meter can be used i.e. squares: 1 m by 1 m. It turned out that children knew from various experiences the notion of *a square meter*. They knew that it is used, among others, to describe the size of a room — what they may know from their parents' discussions. When asked what "a square meter" is, the pupils answered that is a square (1 m by 1 m) and not the one with the perimeter of 1 m. Thus, a unit used for measuring the area of a figure appeared (for children it is the interior not the boundary of a figure). We were able to define other unit squares: 1 square centimetre (and its abbreviation 1 cm^2), 1 square millimetre (1 mm^2), 1 square decimetre (1 dm^2). We carried out a discussion on which of these units can be drawn in their exercise books and which on the football field. The children verified if it was possible to measure rooms with the squares cut out from the chocolates in the previous lesson. Based on this we came to a conclusion in which room the flooring will be the most expensive by stating which of the rooms is the biggest. We calculated also the perimeter of each rectangle. **We adopted a definition that the area of a figure equals the number of unit squares included in the figure**; the area of a figure characterizes its size.

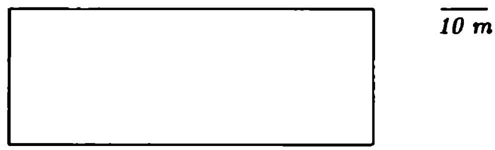
In the next lessons we prepared posters representing:

a) figures of the same area and different perimeter and shape;

b) figures of different area and different perimeter.

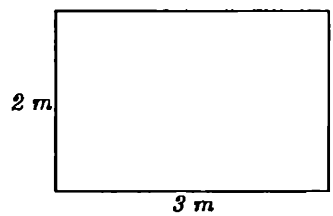
Then we went on to problems connected with the area of a rectangle. In order to control children's understanding of area and perimeter I prepared the following tasks:

1. *Here is Mr Nowak's plot:*



He is going to enclose it with wire netting. How much wire netting will he need? Is his plot bigger then Mr Kowalski's plot of which the measurements are: 80 m by 30 m? Which of them will use more wheat assuming both plots are sown with?

2. *The picture below represents the floor in Ms Zosia's bathroom:*



Miss Zosia is going to lay rose tiles on the floor and face the walls, up to 1 m height, with the colourful ones. Help Miss Zosia calculate how many rose and colourful tiles she will need. Workmen told her to leave out the door and the windows.

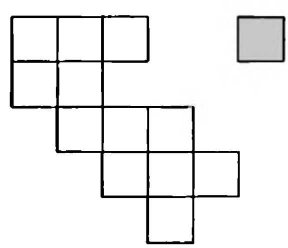
Most pupils did not have difficulties in calculating adequately. They associated correctly the instructions with the area or perimeter of a rectangle. Only in few cases the children still referred to the perimeter of a figure while defining which plot is bigger. However, on recalling previous experiences, they were able to realize their mistake.

In the next few lessons, while treating of the area of a rectangle and solving problems with realistic contexts and contrasting the area with perimeter, pupils showed good understanding of the subject. Furthermore, I noticed that tasks of this type are not only more interesting for them but also easier than the traditional ones. The pupils did not confuse the notion of area with perimeter. They used adequate units and calculated correctly. However, calculating the area of a rectangle consisted in dividing it into unit squares and counting the number of squares, despite knowing the lengths of its sides, they did not use the formula to calculate the area.

The results of the experiment ([1]) mentioned before also confirm good advantage in using activity methods. The authors of the experiment state that these methods are beneficial mainly to weak children and not much helpful to more gifted pupils. However, the comparative analysis revealed that the number of wrong operations in the traditional class turned out to be four times higher then in the class termed as new. Weaker pupils in the new class carried out well almost all operations, while in the traditional class only 50% of the completed tasks were solved correctly. The pupils in the new class did not confuse the carried out operations as often as in the traditional one.

Finally, a few observations made during the lessons.

1. Generally, the children do not have difficulty in drawing figures of any shape and a given area, but the square 9 cm^2 in area was represented as a square of a side 9 cm. We had to divide this square into unit squares and count their number. Still, some pupils did not realize the mistake and drawing an adequate square proved difficult for them. A task of this type calls for paying attention not only to how big the figure is but also to its shape. It would be interesting to see pupils' reaction asked to draw a rectangle with the area of 9 cm^2: would a square arise?
2. Similar difficulties in preserving the adequate shape of a figure appeared in a task: ***Calculate the area of a figure, adopting the black square as a unit, and then draw a rectangle with the same area.*** A pupil drew on squared paper.

   Tasks of this type are difficult for pupils (for many of them very difficult) as they combine geometrical and metrical aspects.
3. Perception of a row-column arrangement of figures is difficult for some children ([6]) both drawing and calculating the number of squares in the whole figure. The relation of the area of a rectangle to the arrangement of unit squares for example in 5 rows of 7 element columns is not obvious for all pupils, at first. Pupils can act differently when they know the lengths of the sides (multiplying these numbers) and otherwise when it comes to finding the are of a cross-ruled rectangle. They often count each square one after the other without using the multiplication.
4. The way of calculating the area of a rectangle and understanding of the formula $S = a \times b$ is conditioned by the way the multiplication concept

was formed in primary classes (if the product 4×7 equals 3×7 plus 7 — the so called linear aspect, or 4 rows of seven elements).

5. Strong tendency to measure the perimeter of a figure while assessing their size, evident at the beginning of pupils' work, may by connected with the order in which these topics are discussed in class. Generally, we teach how to calculate the length of a broken line and the perimeter of a figure, and then the are of a figure is introduced. This raises the question if these topics often occurring just one after the other do not contribute to the difficulties in understanding of the area of a figure, in particular of a rectangle.

The experiences described above reveal difficulties, which arise at the beginning of the concept forming process of the area of a figure. The process will be continued in the following years. Now the foundations have been laid down, how strong — further issues connected with the area of a figure will show. Concept forming is not a single act but rather a long-term process ([5]). For me personally it was a reflection on pupils' understanding, further realization that issues, obvious for a teacher, may be troublesome for the pupils. We do not know much about what they understand by the area or perimeter of a figure as the measure of its size. I realized how complicate the process of familiarizing with a concept is and arriving at its mathematical sense.

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