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About practical exercises taken from real life

Abstract. Review of bibliography. Contemporary requirements in mathematical teaching. Classification of exercises occurring in mathematics school books. Analysis of selected primary and secondary school books considering the use of school mathematics in the surrounding reality. Tentative elaboration of practical exercises.

In this paper we deal with practical exercises connected to daily life. Particularly with those that rise on the ground of problem situations and are problematic themselves, either because of the mathematical solution model or because of post-mathematical problems. Situations of interest to us (practical exercises) contain hidden or overt quantitative interdependencies.
Examples of practical exercises and their working up methodology.

Example no 1. (Multilevel exercises)
25 meters of ribbon, 0.1 mm thick, has been strictly spooled over cartoon tube, and made a shaft of 10 cm in diameter. What is the diameter of the tube? It is one of many exercises which can be found in school-books for mathematics both in primary and secondary school. Basing on this example we will try to show the way how to make the exercise more practical. We will make an analysis to this end according to the below presented items (1, 2, 3, 4).

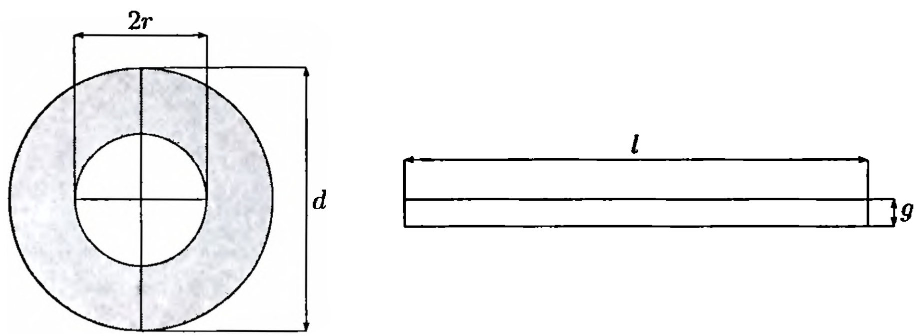
1. Exercise characteristic
It is a pseudo-practical verbal exercise, partly mathematical. A question placed in the exercise is wrongly formulated. In practice, nobody asks for a tube diameter which can be measured approximately. Many data are omitted and a "critical student" can have doubts with them. The said exercise is a good substance for developing pupil's open attitude against a real situation not fully known.

2. Making an exercise more practical by formulating additional questions
When discussing the problem with pupils, it is possible to formulate the following questions: example No 2: Whether layers are placed exactly one over another? Whether winding of a ribbon is finished at a place where winding was started? Whether full winding was not completed because of lack of part of the ribbon? What was the length of lacking ribbon in the said wind? How big was

an error in calculation resulting of it? Is it possible to ask another question in this exercise? What are the data in this exercise? What is searched? What additional data should be involved in the solution? What should be omitted in calculations? What is the simplest solution? What, and what kind of subjects, the exercise is connected to? How to present verbally and on a drawing the necessary data and result and what kind of symbols should be assumed to mark them? How to describe the connection between data and result using variable quantities? And other.

3. Analysis of the situation presented in the exercise

Length and thickness of the ribbon is given (this can be presented as a rectangle on a picture). A tube diameter is being searched. A tube can not be made of too thin a cartoon to avoid squeezing during winding a ribbon on it, it also cannot be made of too thick a cartoon because its thickness should be omitted while calculating. Omitted data make exercise structure simpler and can be neglected in this solution. The simplest solution can be found by measuring the tube's diameter. The data are presented both verbally and on a drawing, as well as marked with symbols used in mathematics and physics.



Data:
Ribbon length — l
Ribbon thickness — g
Shaft diameter — d

Searched:
Tube diameter $2r$ =?

Tube thickness to be neglected.

Figure 1.

In this way we have received a simplified scheme (model) of the situation presented in the exercise, directed towards a searched solution model.

4. Synthesis of a simplified scheme and construction of a mathematical model

Below we present connections between data and searched result using variables.
Area of a ring: $P = \pi\frac{d^2}{4} - \pi r^2$.
Area of a rectangle of length "l" and width "g": $P = l \cdot g$.
Comparing the areas we receive: $l \cdot g = \pi\frac{d^2}{4} - \pi r^2$ as a mathematical model, in

which $l \cdot g = \pi\frac{d^2}{4} - \pi r^2$ is a relation between parameters of this situation.

After completing all necessary conversions we obtain a solution model for this exercise:

$$\pi r^2 = \frac{\pi d^2}{4} - l \cdot g,$$

$$r^2 = \frac{\pi d^2 - 4l \cdot g}{4},$$

$$r = \sqrt{\frac{\pi d^2 - 4l \cdot g}{4}}.$$

After substituting concrete data the approximate radius is 4.13 cm.

If we change a formula of the exercise as in example No 2 and put another question, for example: How many meters of ribbon of thickness 0.1 mm should be tightly winded up over cartoon tube of approximate diameter 8.26 cm in order to receive a shaft 10 cm in diameter? — the mathematical model will not be changed. The simplest solution of this exercise can be received by unwinding the shaft and measuring the ribbon's length. During unwinding of the shaft we can observe that unwinding of one coil reduces the shaft's diameter of 0.2 mm, whether coils are placed exactly one over the second or not, etc.

Simplified scheme of data and searched presentation both verbally and on a drawing in the new situation is analogous to the first scheme. The difference is only that in the first case we are searching the tube diameter and in the second one the length of the ribbon.

Constructing an example solution of the exercise we are starting with the same dependencies:

$$l \cdot g = \pi\frac{d^2}{4} - \pi r^2.$$

Determining "l" needs conversion of a formula to a form:

$$l = \pi\left(\frac{d^2}{4} - r^2\right) \cdot \frac{1}{g}.$$

Composition of a mathematical solution in the first case needs different knowledge than in the second one. As to the formula $r = \sqrt{\frac{\pi d^2 - 4l \cdot g}{4\pi}}$, pupils need to know operations on roots; as to the formula: $l = \pi\left(\frac{d^2}{4} - r^2\right) \cdot \frac{1}{g}$ operations on powers. Therefore the choice of formulation of a concrete question should take into account that pupils' mathematical skills needed for its the solution depend on the formulation.

Learning to solve practical problems every-day situations is a process. Acquired knowledge should be used in new situations, which are yet unknown. Pupils should enter this learning process step-by-step, starting from very small "steps", for example making analysis of a simple para-mathematical situation, simple mathematical dependencies, open or closed, formulated in natural language, rising questions, data valuation, exposing abstracted quantitative dependencies or other, formulation of an exercise against the background of a situation. Choosing a ready model and adopting it to the conditions of formulated exercise or creation of a new model structure exceed the pupils knowledge and skills. The pupils' activities mentioned above should strictly be tied to currently taught subjects. School-books for mathematics do not prepare them for solving practical problems (from real life).

In our opinion pupils should also be confronted with every-day life problems, which are not presented in school-books, for example:

Example No 2. "Mother wants to paper her kitchen and corridor. How much money should she prepare for it, if a wall paper is being sold in a shop in full rolls only?"

Analysis of the above mentioned situation will be presented below. The way of formulation of a mathematical exercise on the background of certain situation and how to build a mathematical model will also be presented.

- It is a practical situation. It does not contain open data, neither quantitative nor qualitative or valuable.
- Formulating a mathematical exercise on the ground of a certain situation the pupil must be able:
  - to plan paper-hanging that means to decide which walls will be totally covered and which ones only partially, having in mind doors, windows, ceramic tails on the walls, radiators, permanently fixed furniture, window recesses or something more;
  - to measure up proper elements of the kitchen and corridor;
  - to remove wasted plaster, paint or old wallpaper from the walls;
  - to get information about prices of wall paper, glue, plaster, brushes etc.;
  - to chose necessary materials, their quantity and prices.

In an exercise formulated by himself, a pupil should quote the following: size of areas to be papered, prices of materials to be bought, and to formulate a question or instruction.

Mathematical model will describe a way of paper-hanging costs calculation using simple arithmetical rules. A solution can be reached using approximate data. Description of such process can be presented on a general diagram:

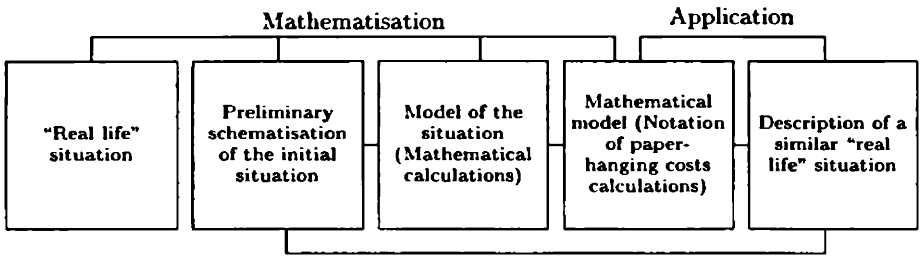


Figure 2.

Initial schematization in this case, which would allow formulation of a mathematical exercise containing data and relations between data and results, should be as follows: paper-hanging planning up, collection of information and choosing necessary materials. In effect we receive a simplified situation scheme which can be used as a source for further study. An elaborated situational model (exercise) is transformed into a mathematical model, in which relations and peculiarities are described in the mathematical language, which next can be used for description of the "real life" situation.

According to R. Pawlak, such application of mathematics is rather pseudo-application, which quite often have nothing common with "genuine" application of mathematics in life.

Teaching mathematics should, however, start from something. For the beginning let it be just a primitive application of mathematics in situations close to a child.

- Difficulties in modelling

School books contain mostly examples of mathematics application in physics and mechanics. Present didactical literature, particularly conceptions of the teaching goals, suggest introducing applications of mathematics in natural sciences as well as economical, humanistic and every-day life problems. Present school mathematics is not good enough for modeling situations taken from real life; the essence of the difficulty is not only in the complex character of processes and phenomena, but also in the variety of technical possibilities of measuring.

- Model

In ordinary language a model means something being a copy of an original. In logic-mathematical sense a model means a structure $(A, R_1, \ldots, R_k, C_1, \ldots, C_m)$ composed of a certain set A as a field of the model, relations $R_1, \ldots, R_k$ between elements of the set A, and certain objects $(C_1, \ldots, C_m)$ belonging to the set A (Borkowski, 1970).

We use to say that a certain model is adequate to a certain real situation or process with regard to its chosen characteristics when it describes it properly (quantitatively and qualitatively, with reasonable accuracy with regard to the said characteristics.

Characteristic of practical exercises:

- practical situations (exercises) binding mathematics with other subjects or every-day life have realistic context, contain secret quantitative, qualitative data, questions or instruction;
- practical situations (exercises) are like this, which allows for: taking independent decisions, rising questions and instructions, collecting information, building preliminary schemes (models) simplifying and exposing chosen relations and characteristic peculiarities, with the help of drawings or verbally, in categories and language of the field the problem comes from, processing preliminary schemes (models) into mathematical models through description of relations and characteristic peculiarities in the mathematical language with which it can be studied;
- practical situations (exercises) help realisation of targets put for the teaching by mathematics and life, i.e. teaching not only mathematics but also the ability to cope with complicated present reality;
- practical situations (exercises) can be classified taking into consideration the degree of complexity of the mathematical model, as well as a context: physical, technical, natural, economical, humanistic and of real life;
- practical situations (exercises) can be classified taking into consideration exercise type: training (simple application of a theory), problematic. Classifications of practical exercises are not disjoint; different types of exercises can belong to the same category.

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