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Direction field of fingerprint image and its optimization with the help of relaxation procedures

Abstract. Direction field building has an exclusive importance for common process of the classification analysis in up-to-date automatic fingerprint identification systems. All characteristics of this analysis strongly depend on direction field reliability. That's why effective technologies development for the optimization of direct field is the actual problem. In this work we pay attention to the smoothing procedures of relaxation type, which use the gradient disbalance minimization principle.

In general process of the fingerprint image (FI) classificational analysis (CA), where $F = \{f(x,y)\}_{(x,y)\in \mathbf{I}_1\times\mathbf{I}_2}$ is an intensity (brightness) at the (x,y) point; $\mathbf{I}_s = \mathbf{Z}_{N_s}$ $(s = 1,\ 2)$; N_1, N_2 are respectively x– and y–dimensionalities of F FI; $\mathbf{Z}_m$ is a notation for the $\{0,\ 1, \dots,\ m-1\}$ set (m is a natural number); a fundamental part is taken by the gradient field $\Delta = \{\delta(x,y)\}_{(x,y)\in\mathbf{I}_1\times\mathbf{I}_2}$ forming [1]. Here $\delta(x,y)$ is a slope angle of a tangent carried out to the ridge or the cavity at (x,y) point toward the x–axis; for the certainty further assume that $\delta(x,y) \in [-\frac{\pi}{2}; \frac{\pi}{2})$. All of the most important qualitative characteristics of FI CA and particulary the correctness of type detecting and pattern orientation, are strongly depend on the Δ field authenticity. In the light of the aforesaid, the effective direction field optimization technologies development is an important and extremely urgent problem of CA.

Fundamental part in the proposed method of the direction field correction is taken by so-called gradient unbalance, which for simplicity we will calculate on the square apertures with the the side length equal to $L = 3$ points. The gradient unbalance for the element (u,v) is determined either by the formula

$$\mu_{u,v} = \left| \sum_{(x,y)\in\mathbf{A}^\circ(u,v;3\times3)} p_{u,v}(x,y)\, |\delta_{x,y} - \delta_{u,v}|_\pi^- \right| \tag{1}$$

or by the formula

$$\mu_{u,v} = \sum_{(x,y)\in A^\circ(u,v;3\times3)} p_{u,v}(x,y)\left||\delta_{x,y} - \delta_{u,v}|_\pi^-\right|, \qquad (2)$$

where $A^\circ(u,v;3\times3) = \{\forall(u+i,v+j)|i,j \in Z_3^-\}\backslash(u,v)$ is a square aperture with missing center (u,v) and side length $L = 3$ ($Z_3^- = Z_3^-(0)$); $p_{u,v}(x,y)$ is weight of the aperture element (x,y) that depends on aperture "direction" $d_{u,v} = |\rfloor 4\delta_{u,v}/\pi\lfloor|_4$; $\rfloor x\lfloor$ is a notation for an integer nearest to x formed according to the rule

$$\rfloor x\lfloor = \begin{cases} \lfloor x\rfloor, & \text{if } x < \lfloor x\rfloor + 0,5; \\ \lceil x\rceil, & \text{if } x \geq \lfloor x\rfloor + 0,5, \end{cases}$$

and $|x|_a$, $|x|_a^-$ are notations for respectively remainder and symmetrical remainder obtaining from division x by a, which they are determined by the ratios $x = |x|_a + \lfloor x/a\rfloor \cdot a$ and $x = |x|_a^- + \rfloor x/a\lfloor \cdot a$ ratios for any real numbers x and $a > 0$.

Let us introduce the concept of the π-module segments for the angle set from the interval $[-\frac{\pi}{2};\frac{\pi}{2})$. Consider the arbitrary numbers A and B from $[-\frac{\pi}{2};\frac{\pi}{2})$. Then the π-module numerical segment $[A;B]_\pi^-$ is a notation for the set

$$[A;B]_\pi^- = \begin{cases} [A;B], & \text{if } A \leq B; \\ [-\frac{\pi}{2};B]\cup[A;\frac{\pi}{2}) = [-\frac{\pi}{2};\frac{\pi}{2})\setminus(B,A), & \text{if } A > B. \end{cases} \qquad (3)$$

The open and half-open π-module intervals are defined by the same way.

The smoothing out relaxation procedures that realize the gradient unbalance minimizing (absolute or threshold-based) in some forms, compose natural and very effective basis for conducting required operation. Usually these procedures achieve a segment-by-segment correction of directions $\delta_{u,v}$, coordinating with the gradient characteristics $\delta_{x,y}$ on $\{F_{x,y}\}$ segment sets, represented by some localities (or their parts) of the $F_{u,v}$ segments. Naturally, it should be considered that the direction $\hat{\delta}_{u,v} = \hat{\delta} \in [-\frac{\pi}{2};\frac{\pi}{2})$ reaching the minimum to the utilized unbalance characteristic of the type (2), or (2) is an optimum result of the executed correction for $F_{u,v}$.

First let us examine the case, when for the image's optimum gradient field procedure synthesis the base characteristic is considered as unbalance of form

$$\mu_{u,v}(\delta) = \left|\sum_{(x,y)\in A(u,v)} p_{u,v}(x,y)|\delta - \delta_{x,y}|_\pi^-\right|, \qquad (4)$$

where $\delta \in [-\frac{\pi}{2};\frac{\pi}{2})$; $A(u,v)$ is an aperture, which corresponds to the $(u,v) \in I_1^{(1)} \times I_2^{(1)}$ point; $\{p_{u,v}(x,y)\}_{(x,y)\in A(u,v)}$ is the selected collection of weights.

As it may be shown, the unbalance characteristics (1), (2), and (4) are periodic functions with π period. Let us write down (4) in the following equivalent form

$$\mu_{u,v}(\delta) = \left| \sum_{(x,y)\in A(u,v)} p_{u,v}(x,y)(\delta - \delta_{x,y} - \rfloor(\delta - \delta_{x,y})/\pi\lfloor \cdot \pi) \right|$$
$$= \left| \delta \sum_{(x,y)\in A(u,v)} p_{u,v}(x,y) - \sum_{(x,y)\in A(u,v)} p_{u,v}(x,y)\delta(x,y) \right. \qquad (5)$$
$$\left. -\pi \sum_{(x,y)\in A(u,v)} p_{u,v}(x,y) \rfloor(\delta - \delta_{x,y})/\pi\lfloor \right| .$$

For obtaining the angles $\hat{\delta}$, with which the characteristic $\mu_{u,v}(\delta)$ reaches the smallest possible value, that is $\mu_{u,v}(\hat{\delta}) = 0$, it is necessary to require sub-modular expression (5) to be zero. We obtain

$$\hat{\delta} = \overline{\delta} + \pi \sum_{(x,y)\in A(u,v)} P_{u,v}(x,y)K_{u,v}(x,y), \qquad (6)$$

where

$$\overline{\delta} = P_{u,v}^{-1} \sum_{(x,y)\in A(u,v)} p_{u,v}(x,y)\delta_{x,y} = \sum_{(x,y)\in A(u,v)} P_{u,v}(x,y)\delta_{x,y}; \qquad (7)$$

$$P_{u,v}(x,y) = p_{u,v}(x,y)/P_{u,v};\ P_{u,v} = \sum_{(x,y)\in A(u,v)} P_{u,v}(x,y);$$

$$K_{u,v}(x,y) = \rfloor(\hat{\delta} - \delta_{x,y})/\pi\lfloor. \qquad (8)$$

If some $\hat{\delta}$ satisfy equality (7), i.e. the unbalance (4) is zero ($\mu_{u,v}(\hat{\delta}) = 0$), then any element of $\{\hat{\delta} + k\pi\}_{k\in \mathbf{Z}}$ set is posessed by the same property. Thus, it suffices to be limited to the search of $\hat{\delta}$, for that belonging to any interval of π length, for example, to $[0,\pi)$ interval, or $[-\frac{\pi}{2};\frac{\pi}{2})$ for our case.

Since $\hat{\delta}, \delta_{x,y} \in [-\frac{\pi}{2};\frac{\pi}{2})$, that $\hat{\delta} - \delta_{x,y} \in (-\pi;\pi)$. Therefore values of (8) can lay within the $\mathbf{Z}_3^- = \{-1;0;1\}$ set only, namely

$$K_{u,v} = \begin{cases} -1, & \text{if } \delta_{x,y} \in [0;\frac{\pi}{2}) \text{ and } \hat{\delta} \in [-\frac{\pi}{2};-\frac{\pi}{2}+\delta_{x,y}); \\ 0, & \text{if } \delta_{x,y} \in [0;\frac{\pi}{2}) \text{ and } \hat{\delta} \in [-\frac{\pi}{2}+\delta_{x,y};\frac{\pi}{2}), \\ & \text{or if } \delta_{x,y} \in [-\frac{\pi}{2};0) \text{ and } \hat{\delta} \in [-\frac{\pi}{2};\frac{\pi}{2}+\delta_{x,y}); \\ 1, & \text{if } \delta_{x,y} \in [-\frac{\pi}{2};0) \text{ and } \hat{\delta} \in [\frac{\pi}{2}+\delta_{x,y};\frac{\pi}{2}). \end{cases} \qquad (9)$$

In this case, as can be seen from (9), at each fixed point (x,y) of the $\mathbf{A}(u,v)$ aperture the range of $K_{u,v}(x,y)$ values can include only two numbers: -1 and

0 in the case of $\delta_{x,y} > 0$, or 0 and 1 in the case of $\delta_{x,y} > 0$, moreover equality $K_{u,v}(x,y) = -1$ is possible only with $\hat{\delta} < 0$, and equality $K_{u,v}(x,y) = 1$ is possible only with $\hat{\delta} > 0$. ¿From the aforesaid it follows that for any $\hat{\delta}$ being a solution of the equation (9) all the terms of $\sum_{(x,y)\in \mathrm{A}(u,v)} P_{u,v}(x,y)K_{u,v}(x,y)$ sum, which are non-zero (if any), are either negative or positive. Because of the noted circumstance, it is possible to find easily all the $\hat{\delta} \in [-\frac{\pi}{2}; \frac{\pi}{2})$ points of the unbalance (4) minimum using the formulas (6)–(9). The following theorem ensuing from that outlined above gives the appropriate structural method of this problem's solution.

THEOREM 1

Consider δ_{-r_-}, $\delta_{-r_-+1}, \ldots$, δ_{-1} and δ_1, $\delta_2, \ldots$, δ_{r_+} are nondecreasely respectively regulated negative and positive elements of the $\{\delta_{x,y}\}_{(x,y)\in \mathrm{A}(u,v)}$ angle set; r_- and r_+ are total angle quantities with corresponding signs ($r_- \geq 0$; $r_+ \geq 0$; $r_- = 0$ indicates the absence of the first of the sequences in question, and $r_+ = 0$ indicates the absence of the second one), and consider P_{-r_-}, $P_{-r_-+1}, \ldots$, P_{-1} and P_1, $P_2, \ldots$, P_{r_+} which is the sequence of weights from the $\{P_{u,v(x,y)}\}_{(x,y)\in \mathrm{A}(u,v)}$ collection, corresponding to the regulated negative and positive directions. Then all the zero points of $\hat{\delta} \in [-\frac{\pi}{2}; \frac{\pi}{2})$ (that are minimum points of the unbalance (4)), are contained by the elements of $\hat{\Delta} = \overline{\Delta} \cup \hat{\Delta}^- \cup \hat{\Delta}^+$ set, where

$$\overline{\Delta} = \begin{cases} \{\overline{\delta}\}, & if \;\; r_- r_+ \neq 0 \;\; and \;\; \overline{\delta} \in [-\frac{\pi}{2} + \delta_{r_+}; \frac{\pi}{2} + \delta_{-r_-}), \\ & or\ if \;\; r_- r_+ = 0; \\ \varnothing & in\ other\ cases; \end{cases} \tag{10}$$

$$\begin{aligned} \hat{\Delta}^- = &\left\{ \forall \hat{\delta}_{-i} = \overline{\delta} - \pi \sum_{j=0}^{i-1} P_{r_+ - j} \,\middle|\, \hat{\delta}_{-i} \in \left[-\frac{\pi}{2} + \delta_{r_+ - i};\ -\frac{\pi}{2} + \delta_{r_+ - i + 1} \right); \right. \\ &\left. i = \overline{1, r_+} \right\}; \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{\Delta}^+ = &\left\{ \forall \hat{\delta}_{i} = \overline{\delta} + \pi \sum_{j=0}^{i-1} P_{-r_- + j} \,\middle|\, \hat{\delta}_{i} \in \left[\frac{\pi}{2} + \delta_{-r_- + i - 1};\ \frac{\pi}{2} + \delta_{-r_- + i} \right); \right. \\ &\left. i = \overline{1, r_-} \right\}. \end{aligned} \tag{12}$$

The $\overline{\delta}$ direction which means to be average for the $\mathrm{A}(u,v)$ aperture is calculated according to the formula (7); in (11) and (12) we assume $\delta_0 = 0$.

According to the formulated theorem, for obtaining unbalance $\mu_{u,v}(\delta)$ zero points $\hat{\delta}$ (see (4)) in accordance with (10)–(12) is sufficient to form $\overline{\Delta}$, $\hat{\Delta}^-$ and

$\hat{\Delta}^{+}$ sets, and it is necessary to mean that if $r_{+} = 0$ then $\hat{\Delta}^{-} = \oslash$, and if $r_{-} = 0$ then $\hat{\Delta}^{+} = \oslash$.

References

[1] V. U. Gudkov, A. A. Kolyada, A. F. Chernyavski, *New method for detecting the gradient field of fingerorint images*. Methods, algorithms, and versatile information technologies softvare for authomated fingerprint identify systems: The scientific articles anthology, Minsk, Universitetskoe, 1990 (in Russian).

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