Annales Academiae Paedagogicae Cracoviensis
Folia 16 Studia Mathematica III (2003)

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Wallas - Polya - Hadamard and creativity

Abstract. Comparison of theories of G. Wallas and G. Polya with school practice

Wallas:

1. Preparation (hard work on a problem).
2. Incubation (concious preparation is changed into unconscious mechanism of searching for the solution).
3. Illumination (an idea emerges into a conscious one).
4. Verification of the right idea.

Polya:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back.

Observation of the process of problem solving on some examples.

1. Introduction

Herman Ludwig Helmholtz (1821 - 1894), the great German physicist and mathematician, speaking in 1891 at a banquet on his seventieth birthday, described the way in which his most important thoughts had come to him. He said that after previous investigation of the problem "in all directions ... happy ideas come unexpectedly without effort, like an inspiration. So far as I am concerned, they have never come to me when my mind was fatigued, or when I was at my working table ..."

According to *Graham Wallas*, the English psychologist, Helmholtz gives here three stages in the formation of new thoughts.

The first in time stage is the PREPARATION, during which the problem is, "investigated in all directions". As the second comes the stage during which we are not consciously thinking about the problem. *Walles* calls this stage INCUBATION. The third stage, consisting of the appearance of the "happy idea" together with the psychological events which immediately preceded and accompanied that appearance is ILLUMINATION. These stages are followed

Supported from the grant GAČR 406/02/0829.

by a period of VERIFICATION, in which both the validity of the idea is tested, and the idea itself is reduced to exact form.

Henri Poincaré (1854 - 1912), French mathematicians and physicist, writes in his description of the Verification stage, "that unconscious work supplies ready-made results of a lengthy calculation in which we have only to apply fixed rules. All that we can hope from these inspirations, which are the fruits of unconscious work, is to obtain points of departure for such calculations. As for the calculations themselves, they must be made in the second period of conscious work, which follows the inspiration, and in which the results of the inspiration are verified and consequences deduced. The rules of these calculations are strict and complicated; they demand discipline, attention, will, and, consequently, consciousness." In the daily stream of thoughts these four different stages constantly overlap each other.

Jacques Hadamard (1865 - 1963), distinguished French mathematician, made during World War II in New York his own introspections on the creative process and asked major scientists, mathematicians and artists for their view as well. He reported on some of their insights, including these of the linguist *Roman Jacobson*, the anthropologist *Claude Levi Strauss*, and the mathematicians *George Polya* and *Norbert Wiener*. Perhaps his most famous informant, however, was *Albert Einstein*, who described his own thinking process. In a letter to *Hadamard Einstein* wrote that words seemed to play no role in his mechanism of thought, which instead relied on "certain signs and more or less clear images". According to *Philip Nicholas Johnson - Laird*, who wrote in 1996 the Preface to [5], Hadamard made a cogent case for the existence of unconscious mental processes. According to *Roger Penrose* [7] mental processes of mathematical invention cannot even be modelled computationally.

2. Process of Education

In my opinion, it is important to distinguish three inseparable aspects of mathematical education:

a) *Development of mathematical skills* (mathematical craft, mathematical trade).

b) *Stimulation of mathematical understanding.*

c) *Cultivation of mathematical creativity.*

These aspects should be realized in mathematics education from the very beginning. A single illustration of that idea is given in the 3rd part of my contribution.

The great American psychologist *Jerome Bruner* wrote his famous book *The Process of Education* [2] in 1963 and in 1995 he published the study *The*

Culture of Education [8] which is apparently much more realistic. I would like to quote some ideas from it.

"There are four dominant models of learners' minds that have held in our times. Each emphasizes different educational goals. These models are not only conceptions of mind that determine how we teach and "educate", but are also conceptions about the relations between minds and cultures. Rethinking educational psychology requires that we examine each of these alternative conceptions of human development and reevaluate their implications for learning and teaching.

1. *Seeing children as imitative learners: The acquisition of "know how".*
2. *Seeing children as learning from didactic exposure: The acquisition of propositional knowledge.*
3. *Seeing children as thinkers: The development of intersubjective interchange.*
4. *Children as knowledgeable: The management of "objective" knowledge.*

Real schooling, of course, is never confined to one model of learner or one model of teaching. Most day–to–day education in schools is designed to cultivate skills and abilities, to impart a knowledge of facts and theories, and to cultivate understanding of the beliefs and intentions of those nearby and far away."

The last section of Bruner's quotations is in my opinion also the estimation of our school. We have great difficulties with the development of thinking in mathematics.

The famous American mathematician *George Polya* (1887 - 1985) studied in his book *Mathematical Discovery* [3] these questions at large. His main ideas are:

Teaching is not a science: it is an art. The main aim of teaching is to promote the mind, to learn how to think.

Three principles of learning:

a) Active learning;
b) Best motivation;
c) Consecutive phases. Solving of routine problems, "complex" problems and "creative" problems.

After 20 years Polya returns to these questions in his contribution at the *Fourth International Congress on Mathematical Education*, where he writes: "I repeat with conviction that mathematics promotes the mind, but in fact this is not unconditionally correct. There is a condition: Mathematics promotes the mind provided that it is taught and learned appropriately. I could quote to you examples of mathematics teaching, which is quite good to transmit the intended mathematics facts and its proofs but does nothing to promote the mind."

Thinking is closely connected with problem solving. In case that the processes described by *Wallas*, *Poincaré* and *Hadamard* [5] are typical for problem solving it is clear that we must prepare the appropriate conditions for those processes. One of them is plenty of time and long termed preparation of this effort. Some examples of such "strategic motivation" brought out *Milan Hejný* in his book *Teória vyučovania matematiky* [6].

3. Two examples

The first example is a common school problem for students aged 16 years:

Find the area of a regular dodecagon inscribed in the circle with radius r.

Two solutions of this problem are in Fig 1, the third solution is in Fig 2.

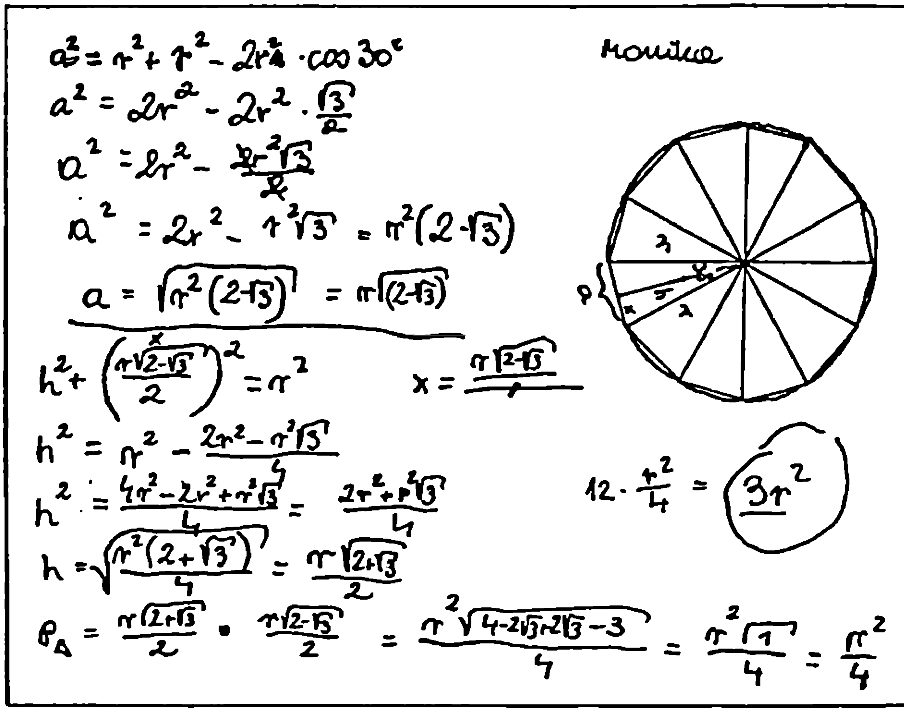
Monika (21 years old) solved the problem correctly, but in a very complicated way. *Vladimír* (22 years) solved the same problem with simplicity and creatively. He saw all coherences and exploited them. *Marek* (22 years) "knows" mathematics, but does not know how to use it. During the solution process he made many blunders. His mathematical culture is very low, his knowledge is formal.

The second problem is for you. I know eight quite different solutions of it. These solutions are simple, but require certain portion of creativity. Send me, please, your solutions of this problem:

Prove: If in a triangle ABC median CS and altitude CP divide angle ABC into three congruent angles then is AC perpendicular to BC.

Conclusions

My belief is that mathematical skills, understanding of mathematics, and creativity are very closely connected to each other and are of use by solution of all problems. These elements of the process of education are feasible to cultivate from the very beginnings of school attendance.



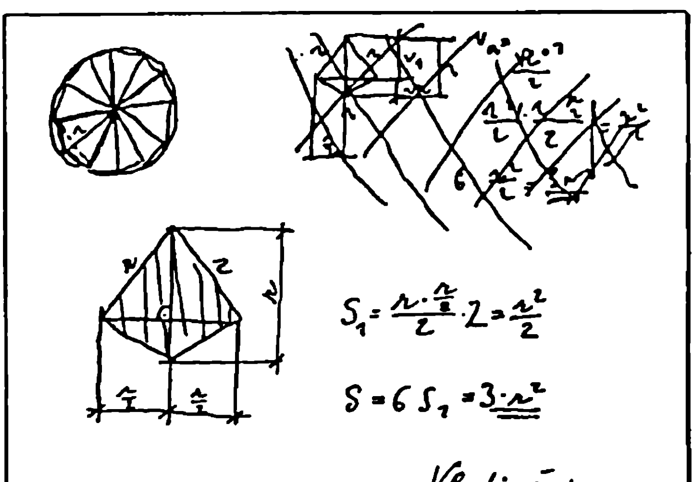


Figure 1.

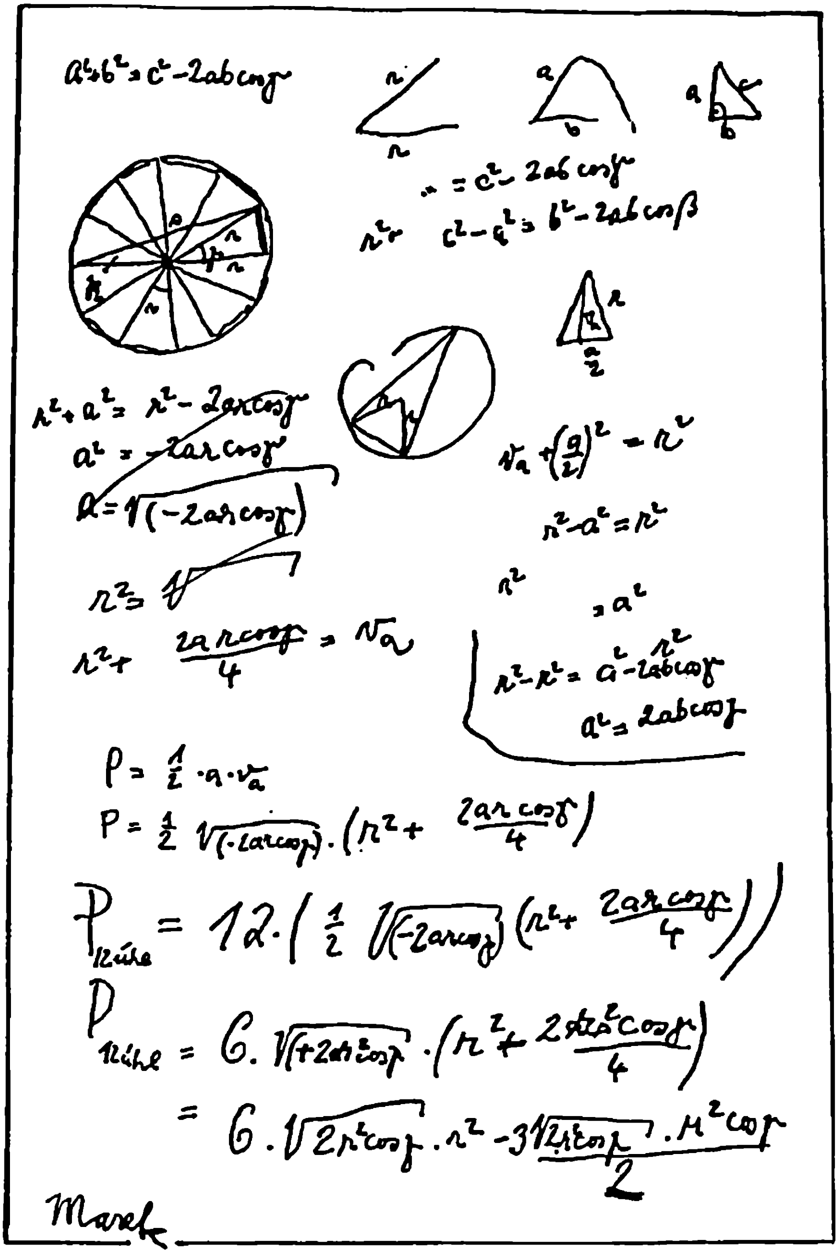


Figure 2.

Appendix

GEORGE POLYA:

TEN COMMANDMENTS FOR TEACHERS

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: The best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but "know - how", attitudes of mind, and the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come — try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once — let the students guess before you tell it — let them find out by themselves as much as is feasible.
10. Suggest it, do not force it down their throats.

Mathematics Discovery II, p. 116

References

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