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Mathematical logic and mother tongue

Abstract. Logical thinking is closely connected to the terms statement and logical evaluation (determination if it is true or false). A sentence is the language expression of thought. It means that logical thinking is closely connected to the mother tongue. The idea of a sentence can be introduced even at the elementary school level as a declarative sentence, which clearly expresses something that can be true or not. If we use one of many options in the sentence the opposite declaration must contain all other options. In the present article sentences about numbers of people and things and their opposites are studied. There are also examples of interconnection between logic and the mother tongue in education at the elementary school.

Logical thinking is closely connected to the terms statement and logical evaluation (determination if it is true or false). A sentence is the language expression of thought. Thus logical thinking is closely connected to the mother tongue: sentence expression and logical evaluation.

The term statement was introduced within an experiment in the 5th classes of the basic schools. As a main tool we used the relationship between mathematics and mother tongue. Students' knowledge of sentence types played an important role.

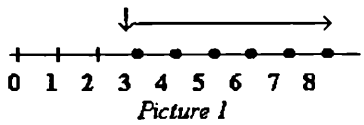
We said to the students that each indicative sentence is considered to be a statement in case it expresses something that can be either true or false. The word "statement" was generally understood by students as memorable sentence of some known or important person, or judgement of a court, or announcement of a group of scientists or specialists etc. They were persuaded that a statement has to be true. We considered this fact and explained that an untrue sentence can be a statement too. Indicative sentence "one plus one is five" is a statement. It is a declaration clearly proclaiming something that is not true. This sentence can be noted down in digits instead of words "$1 + 1 = 5$". The written record is different but oral expression is the same.

Students learned that sentences like "Jump!", "What time is it?" are not statements because they are not indicative. On the other side, sentence "Cat is a multiple of a dog's tail" is not a statement because it does not say anything understandable.

Students discussed the question how to determine true or false value of a statement. They said that it can be determine based on personal experience, literature or internet or information given by a reliable person. Despite those sources we can make a mistake. Let's take history as an example; e.g. the date of letter print invention varies in different sources. It was said that practice is one of the criteria to determine the truth but how to decide, according to practice, about the correct date of the letter print invention.

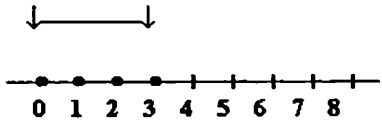
We studied negation of a statement too. If we mark a given sentence by letter q than the sentence "It is not true that q..." is called negation of the statement q. The word negation comes from the Latin verb "negare" which means "to deny". The statement "Prague is the capital of Czech Republic" can be negated as follows "It is not true that Prague is the capital of Czech Republic". Mathematical statements are negated as follows "It is not true that $1 + 1 = 5$". We can see that negation changes true statement into a false one and vice versa. Negation of a statement can be expressed in a shorter version, e.g. negation of the sentence "I have a white sweater" is "It is not true that I have a white sweater", and its shorter version is "I do not have any white sweater". Mistake can appear at this point. Students may negate the statement "I have white sweater" by the statement "I have blue sweater". In this case it is not a negation of the original statement "I have a white sweater". Negation of the statement "$6 > 2$" is "It is not true that $6 > 2$"; shorter version is "$6 \leqslant 2$" not "$6 < 2$". Generally: "If in a statement there is one of many options the negation must contain all other options". In some statements there is stated an amount or estimated amount of persons, things or mathematical objects of a certain quality. Those information are expressed by words: at least one, two, three; just one...; at the most one...; each; all; none. In this domain, students' knowledge of placing zero and natural numbers on the number line is of importance.

at least 3... means 3 and more



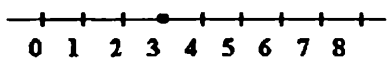
Picture 1

at the most 3... means 0,1,2,3 and not more



Picture 2

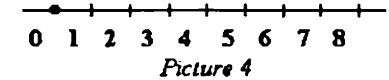
just 3... means 3 not more or less



Picture 3

⇓

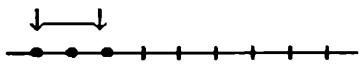
none ... means 0 and not more



Picture 4

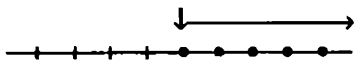
Figure 1.

The above mentioned statements can be negated by the words "it is not true that ..." or we can use the sentence "If in a statement there is one of many options the negation must contain all other options". Then negations of the statements above give numbers of persons or things not marked by black points. Those cases are shown at pictures number 5 – 8 and worded on the right side



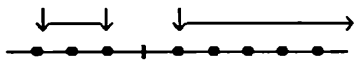
Picture 5

0,1,2 and not more... at the most two



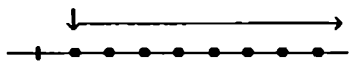
Picture 6

4 and more... at least four



Picture 7

0,1,2 or 4 and more...at the most two or at least four



Picture 8

1 and more... at least one

Figure 2.

Bellow we can see examples of statement negations from mathematical exercise books:

1. "At least four students get mark *A*"; negation is: "It is not true that at least four students get mark *A*" or "At the most three students get mark *A*";
2. "At the most for five days the temperature was bellow zero"; negation is

"It is not true that at the most for five days the temperature was bellow zero" or "For at least six days the temperature was bellow zero";

3. "Just for three days it was raining"; negation is "It is not true that just for three days it was raining" or "At the most for three days or at least for five days it was raining".

References

[1] J. Melichar and col., *Cvičenia z matematiky pre 5. ročník základnej školy*, (Exercises of Mathematics for 5th Classes Basic Schools), Slovenské pedagogické naklad., Bratislava, 1992.

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