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Prospective teachers and mathematical word problems

Abstract. Mathematical word problems and the solving process of them within the framework of the system of educational aims and pedagogical competencies of primary and secondary school teachers. Experience with developing the competence to prepare and solve word problems during teacher training.

1. Introduction

The experience in mathematics teacher training puts stress on the demand for an integrated approach, which emphasizes personal qualities of the prospective teacher. The necessity to gradually change the strategy of teaching mathematics both at the primary and secondary schools and during the university training (see [1]) stands out clearly. So does the necessity to create conditions for wider application of activity forms of teaching and skill–requiring educational techniques. We believe that one of the key fields that could be affected by these changes is the work with word problems. The place and role of word problems in the professional training of prospective teachers of mathematics means not only the ability to solve the problems and the competence to manage the learning process of students when they solve them. It also covers broader concepts of the didactic functions of problems in the educational process, e.g. the ability to intentionally prepare the problems. The ability auto-reflection on these activities (stressed by [5]), is considered an important didactic competence of teachers.

2. The position of word problems and their solution in the teacher training

In our opinion, it is possible (with certain amount of simplification) to distinguish two main phases, in which the prospective teacher's attitude to word problems undergoes.

By solving these problems the students show the level of their acquisition of mathematical knowledge and ability to apply it, often under stress. The and function of word problems as both application of mathematics and diagnostic

tool has been dominant even before the students entered their university training — during the entrance exam or the "prolonged entrance–exam period" in the beginning of the studies (see [4]).

This didactically oriented component of the prospective teachers' training also develops their cognitive competencies. Prospective teachers are supposed to demonstrate their ability to prepare and perform teaching by means of construction of a structured set of problems using parameters set by the curriculum, a textbook, or their own style of teaching. Setting conditions for the development of competencies to creatively apply modified techniques when working with the problems enables the desired shift from acquiring scientific subject competencies of a teacher to acquiring psycho–didactical competencies.

In our contribution we shall try to describe our experience from working with mathematical word problems during the teacher training of prospective primary school teachers

- in the beginning of their university education in the introductory parts of the scientifically mathematical component aimed at the students' solving competence,
- during the seminar on didactics of mathematics in the concluding semesters of studies aimed at the development of the ability to adequately change or prepare mathematical word problems.

3. A contribution to the competence to solve word problems

In the beginning of their university studies our students are given a test with word problems taken from the elementary school subject matter. This tests their ability to solve word problems. In the last few years the following problems have been placed into the test:

1. *Jane had a bag of balls. After she gave half of them to Peter and a third of the rest to Dasha, she was left with 6 balls. How many balls had there been in the bag?*
2. *In the garden there are between 90 and 100 trees. One third of them are apple trees, one quarter of them are pear trees and the rest are cherry trees. How many trees are there in the garden?*
3. *A square bed with a side of 10 m is to be fenced. Certain number of columns had therefore been put into the ground, 2 m apart each. How many columns were needed?*

Conclusions from the analysis of students solutions

Using the Toom's terminology (according to [6]) the above mentioned problems are *word problems* or *real-world problems*. The level of success of the

sample of the 1st year students of elementary school education was low (58% for No. 1, 62% for No. 2 and 73% for No. 3) and it met our expectations. We interpret the results as a confirmation of a long-term state reflecting a very low level of mathematical knowledge and skills of primary school prospective teachers when entering the university. Standard subject matter of elementary school had thus not become a permanent quality of a prospective teacher which is a condition of their concentration on its didactic transformation for pupils and other aspects of education. As Krygowska, in her classic work, already [2] pointed out, the transformation of the text of a word problem into a mental construction of a reader who is trying to solve it is a necessary condition of finding a solution.

Analysis of students' solutions enabled us to distinguish:

- *algebraic solutions* in which one or more equations were used (in problem No. 1 creating and solving a linear equation $\frac{x}{2} + \frac{x}{6} + 6 = x$),
- *arithmetical solutions* following from own idea of the solver, usually an experiment (in problem No. 2 a number $90 < x < 100$ divisible by both 3 and 4 must be found),
- *solutions using visualization* of the structure of the problem by using a graph scheme (especially in problem No. 3 the graphical representation proves a "visual literacy" of the solver).

4. A contribution to the competence to prepare mathematical word problems

When compared to the learning process of the solver, this is a new, qualitatively higher level. Intentional and planned creation of problems according to given rules and parameters which would minimize chance, spontaneity and intuition in the teacher's activity must take into account aims and content of education and anticipate methods and strategies of students' solutions. These competencies do not emerge by themselves, the way to achieve them is long and tedious. We shall give examples of cognitive training during seminars on didactics of mathematics aimed at creating a set of problems by changing a given problem.

From a default situation expressed by a composed word problem of a chosen operational structure, e. g. of the $a + (a + b)$ type:

> *People going on a trip got on two buses. There were 36 people on the first bus and 4 more on the second bus. How many people traveled on both buses?*

We wanted students to create word problems:

- solvable by two operations like the given problem

- of the same operational structure with the context changed,
- of the same context with the structure changed, e. g. $a + (a : b)$,
- with both structure and context changed,
- which had to be solved by using more operations and in which the partial task are interconnected.

Some conclusions from the students' projects

Students often created linguistically poor problems; some of the suggestions were linguistically incorrect, incomprehensible or difficult to understand. The authors often made an arithmetical problem which they later "wrapped" with an invented text which did not often correspond to reality. They underrated the influence of the text itself on the quality of its solution in at least three dimensions:

- *linguistic part of the task* (not only stylistic one but also morphological or syntactic inaccuracy, transgressions of the language norms),
- *semantic aspect* including comprehensibility and unambiguity of the presentation of the word problem; the problems have a low motivational value for the pupils because of their topic or way and form of linguistic expression of their respective components,
- *interpretation of the expected result* — judging the possibility and suitability of the solution with potential use of pupils' own experience, ensuring that the problem is solvable in a given field of numbers (e.g. vain attempts to invent a meaningful variation of the $a - (a + b)$ problem solvable in the field of natural numbers.

It often was that pupils were blamed for errors that occur in their solutions and not students themselves for their lack of skill when preparing and testing the problems.

5. Conclusion

When giving opinions on mathematics and approaches to teaching it, Kuřina in [3] (p. 3) says: "*mathematics means solving problems. This part of mathematical education is very important and when using a set of problems carried out in a suitable way, it can lead to not only acquisition of a desired set of mathematical knowledge but at the same time to teachers who systematically cultivate pupils' ways of reasoning.*"

We consider acquisition of prospective teachers' competencies to solve and prepare word problems as one of the conditions of attaining the state when pupils themselves are motivated to productively and creatively solve and prepare problems. It has been proven many times in real-life situations that when

inventing a task pupils can prove their knowledge better than when solving it. However, we consider these few ideas on how to work with word problems to be only a preliminary training for specific didactic application.

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