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Jaroslav Perný

The space imagination on the cube

Abstract. The contribution presents some results of the space imagination research on the cube carried out on 9-to-12-year old students.

The experiment is based on a task situation called "Tilting a die", in which a student "tilts" the die over its edges in his/her mind, according to a plan. A group of tasks of different types, length and difficulty were composed. This research is the continuation of the before mentioned exercises "Walking on the cube" and "Imagining a cube from its given net". Differences of success between boys and girls, older and younger students and so on were investigated. Some common phenomena occurred in the students' solving strategies. These are for example difficulties in understanding the direction ahead — to the back, the use of movement and the regularity in tilting.

There appear many problems concerning solving space tasks in the teaching of mathematics in various schools. The necessity of searching for methods of improving and advancing space imagination of most of students and pupils is revealed.

That is the reason for me to examine and experimentally test which kinds of tasks for the development of space imagination are applicable in the first grade of basic school. These are the tasks that can be introduced into education as an amusement or warming up for the lesson, without direct connection to the class topic. The effort is made to find tasks that do not require two-dimensional models of space objects; the solving process should be conducted in the mind of the student. I have aimed my research at students 9 to 12 years old.

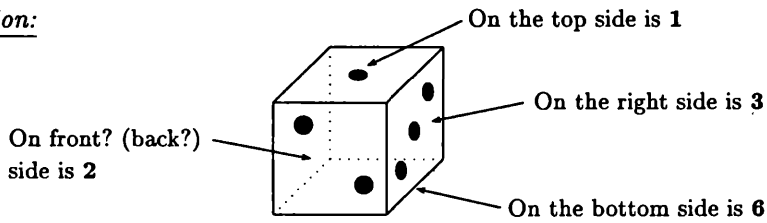
A new set of experiments called "Tilting a die" is presented here, related to the previously presented experiments "Walking on the cube" and "Imagining a cube". The new set was created as an analogy to the set "Walking on the cube". The space imagination in the environment of cube in motion is practiced in both these experiments.

This set of experiments is based on a task situation "Tilting a die", in which the student tilts the cube in his mind over its edges according to the scheme, trying to follow the side on which the cube is lying.

Student's interview with the researcher completed with observation is the method of the research. The initial position of the cube and the terminology

of die sides and movement directions is discussed with the student in advance. After practicing, when the student is allowed to manipulate with the die, follows a task to be solved in the imagination, without manipulation with the die. The student records the task results on the test sheet.

Initial position:

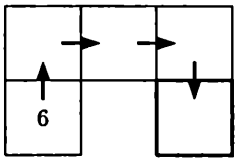


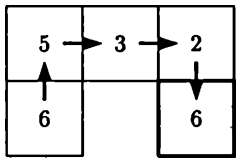
Illustrative example: (practice)

Experimenter says:
"We start with the cube in the initial position. Roll the cube according to arrows on the scheme and continuously record values on the bottom side."

Student: solves and writes the results in the second scheme.

Result:

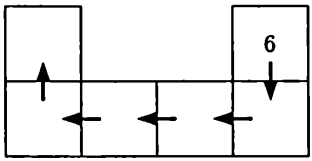




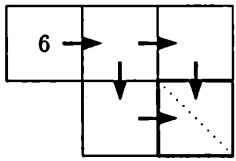
After that the student solves tasks of various type, length and difficulty. The experiment runs undisturbed, communication between student and experimenter is tape recorded. The tape, the test sheet and experimenter's notes are used to complete a detailed protocol.

Examples of tasks:

Task 2:



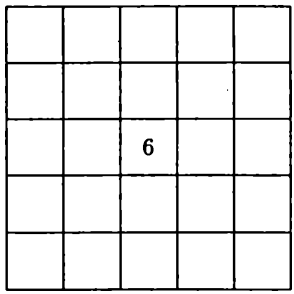
Task 4:



Task 5:

(roll to written values
and mark the arrows to plan.)

6 – 3 – 2 – 1



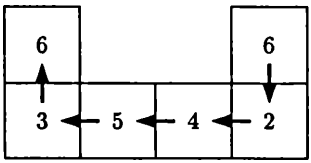
Task 2 examines, how the student manages imaginative "tilting" a die according to scheme, whether correctly, which steps are difficult, how he helps himself, whether he uses the knowledge gained during practicing phase, or whether he has discovered and uses any regularities in tilting.

Task 4 examines, besides the imaginative "tilting", which of two options of different difficulty the student prefers ad whether he finds other regularities in tilting.

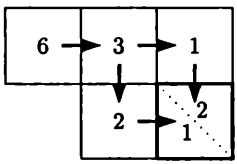
Task 5 is of different (opposite) character and it follows a brief practice with the opportunity to manipulate with the die. Also here it is examined whether the student applies his knowledge from previous manipulations or has discovered and applies other regularities in tilting.

Solving:

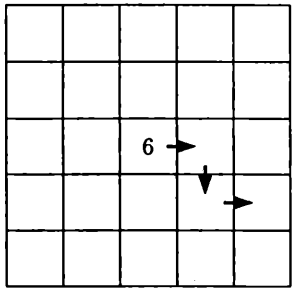
Task 2:



Task 4:



Task 5:



Data gained from test sheets and protocols are further processed into tables and charts.

Determination of the "theoretical difficulty" of steps of the course:

Criteria of comparing results were determined. I tried to evaluate the a

priori difficulty of individual steps of tilting: whether it is the first steps from the initial position (1 pt.), or whether it is one of further steps, or whether the direction of tilting changes (2 pts.) or whether it stays the same (1 pt.), whether there are two changes in direction (3 pts.) or whether the changes occure in the last steps of a longer course of tilting (+ 1 pt.). The score is expressed in % of successfully solved tilts: 1 pt.... 90%, 2 pts... 70%, 3 pts... 50%, etc.

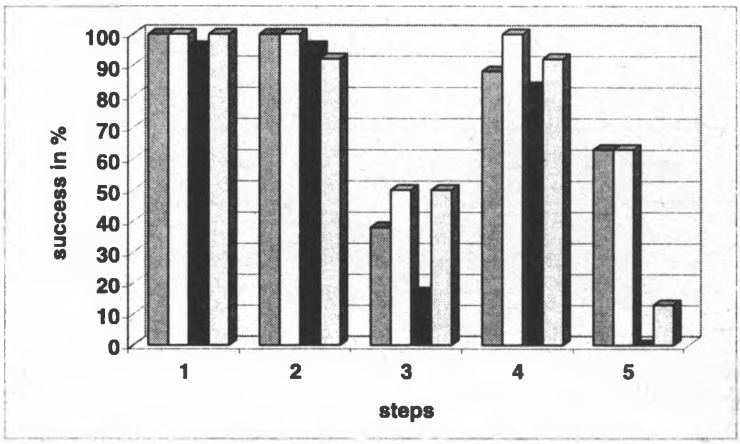
	Class G/B	Theoretical difficulty of step (%)				
	%	90	90	70	70	50
successfully solved tilts	6B	100	100	50	100	17
	6G	100	92	17	83	0
	5B	100	83	50	83	9
	5G	92	100	17	83	0
	4B	100	100	75	100	75
	4G	100	100	75	75	100
	3B	100	100	25	100	50
	3G	100	100	0	100	25

Table 1. Comparison of "theoretical" and real difficulty in Task 2

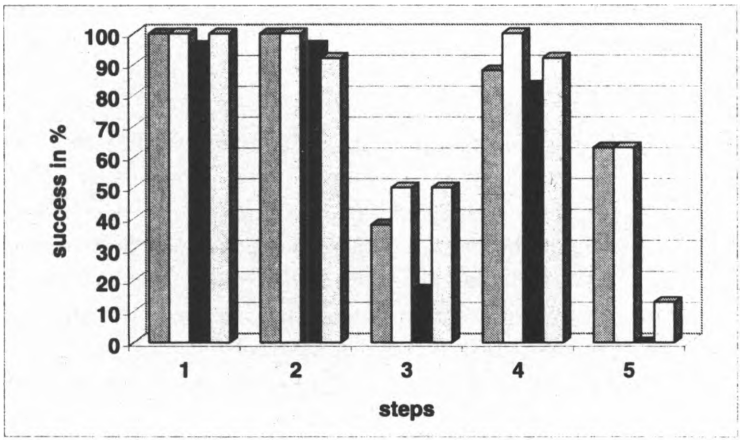
It is obvious from the table, that the 4th step was easier then presumption while 3rd and 5th steps were more difficult then presumption (light or dark gray shading), except for the grade four students that applied more regularities in tilting.

Average success in solving:

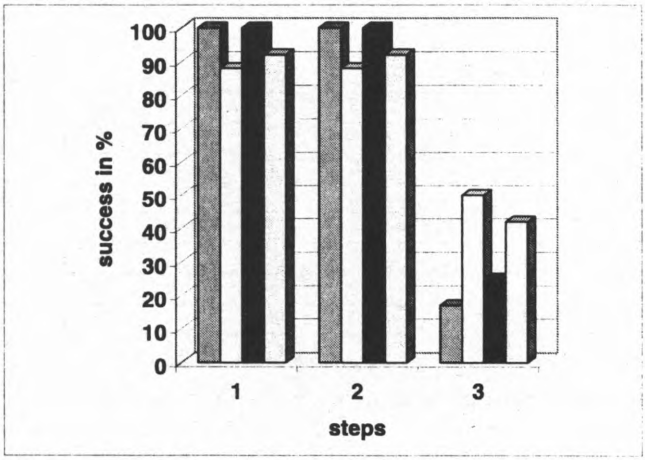
Task 2:



Task 4:



Task 5:



Legend: Younger girls gray, younger boys white color, older girls dark gray, older boys light gray.

Some findings: *(experiments were carried out with students of the 3rd and 4th classes, and also with the 5th and 6th classes).*

It was confirmed that steps with multiple changes in tilting direction and steps at the end of a longer course of tilting were most difficult.

- Boys are generally more successful then girls, in more difficult steps younger students are not less successful then older ones.
- In case of two possibilities of solution, most students chose as the first the simpler one, as the second the more difficult one.

- It appeared that some students during solving the task or checking their solution used regularities discovered during tilting a die, some of them even without bein aware of it (so called "tacit knowledge") (Tab. 2).

  For example:

  - During tilting a die without a change of direction, the sum of values on every other side equals 7 (rule "every other 7").
  - If a reverse change follows a change of direction (even with few inter-steps), the value on the field before the first change and after the reverse change are equal (rule "equal number").
  - A more difficult course of tilting in task 4 is analogical to tilting in "inverse" task 5 (rule "transfer 4-5").
  - In two courses of tilting in tasks 4 the values mutually change from 1-2 to 2-1 (rule "exchange").

Class sex	"Every other 7"			"Equal number"				"Transfer"	Exchange
	Found	"Tacite knowle-dge"	Partly uses	Found	"Tacite knowle-dge"	Partly uses	Uses consis-tently	Found and uses	Found and uses
6B	50	33	83	33	33	33	17	17	17
6G	50	33	66	17		17			
5B	33	17	50	17		17		17	
5G	33		33	17		17			25
4B	50	25	75	100		75			
4G	75		75	100		50			
3B	50		25	75		50			
3G	50		50	100		25			

Table 2. Discovering and applying regularities in tilting (in %)

Other findings common for space imagination in the environment "Cube in motion" *(together with the above mentioned experiments "Walk on the cube").*

- A strong need of performing a motion during mental tilting was proved. Younger students help themselves with both hands, older less prominently with head, and several students combine more movements. Some of them use also voice (Table 3).
- It appeared again that students have problems with understanding the directions "forward — back". For younger ones, the direction forward is from them, for some older ones forward means from them as it is common for us. Some of them were confused and were changing their understanding of direction during the experiment. Additionally, some students present direction forward as down and direction back as up. It is prob-

ably related with their stronger connection to the scheme they use during tilting (Table 4).

Class sex	Hands	Finger	Head	Finger + head	Finger + eyes	Voice
6B	17	33		33	17	17
6G	17	33	17	33		17
5B	50	33	17			
5G	17	50		33		33
4B	33	33		34		25
4G	33	50		17		25
3B	50	33		17		25
3G	33	50		17		25

Table 3. Movement and voice in tilting a die (in %)

Class sex	All usually	Each different	All reverse	Up or down
6B	33		67	33
6G	17	17	66	33
5B	17	17	66	33
5G	0	33	67	67
4B	25	25	50	50
4G	25		75	59
3B	25		75	50
3G		25	75	50

Table 4. Understanding of directions Understanding directions forward — back (in %)

Experiments prove that the development of space imagination can be initialized by appropriate means in the first class of primary school and that it is suitable to include these tasks in the mathematics classes even in cases when geometry is not on the schedule. Not only the natural playfulness of younger students can be used with advantage, but also the similar level of space imagination of boys and girls of this age.

Some of these partial results should be verified on larger number of students to prove their general validity. For some phenomena it is necessary to create other specific sets of experiments that would increase their value as an evidence, and consequently verify them with students.

References

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Technical University in Liberec
Faculty of Education
Hálkova 6
461 17 Liberec
Czech Republic
E-mail: jaroslav.perny@vslib.cz