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Some remarks about pupils' concept of a point

Abstract. Our contribution deals with pupils' conception of some elementary objects of geometry, like a point or a straight line. Pupils (of primary and secondary schools) have often many difficulties with understanding concepts whose phylogenetic and ontogenetic aspects of images differ essentially. Pupils' statements often look like an ignorance or a mistake, but they make evidence of the natural evolution of those concept in pupils.

We conducted interviews with 11-15 years old pupils during the academic year 2001/2002. The article presents some interesting findings about pupils' concept of a point as a geometrical object (in the common school point of view), of the relation "be an element of", and of the density of points on lines.

Pupils were asked the following questions:

1. Let us have the following problem: We have a segment AB. Divide it by means of a point C in the ratio of 2:3. Use scissors and cut the two parts which originated in the division by point C. What will the boundary points of both segments be?

   Is it possible to put a point next to another one so near that nothing is between them?

   How large is a point?

2. We have two segments, one is 5 cm long and the second is 10 cm long. Which segment has more points?

   We have two circles with radius 5 cm and 10 cm. Which circle has more points?

We very often get answers that when we cut a line segment, two segments with endpoints A, C and C, B will form. But a problem remains: Is there only one point C or there are two neighboring points? (Fig. 1)

One opinion of the pupils is that we can cut point C just in the middle, then $C = D$. The more common opinion is that there is a point D near point C and this point D is the endpoint of the second line segment.

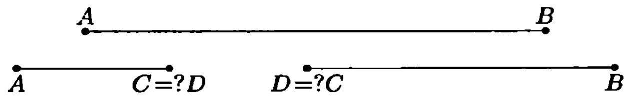

Figure 1.

Children often answer that it is possible to put a point next to another one. It's connected with pupils' imagine of finite size of a point. It corresponds with Democritos' atomism. We can find this position in the theory of J. Sobotka (Czech mathematician, first half of the 20th century) about neighboring points or in the theory of infinitesimally close points in nonstandard mathematical analysis.

We can split pupils into two groups with respect to their opinion of the size of a point. Pupils of the first group think that a point has a certain size and they very often say that the size of a point is 0.1 mm (a relatively big number, because they have met smaller numbers in mathematics and physics). The other group (older pupils, but we can't to say concrete age) thinks that a point has no size.

If a point has a certain size the answer to the question No. 2 clear. The longer line segment or circle has more points than the shorter line segment or circle. But this answer is common also for pupils who do not assign a size to points.

From the interview with girl 14 years old: "*Points are put on a circle like beads*". We can assign points of the bigger circle to points of the smaller circle with the help of a line crossing the center. We found the line ran over between two points of the circle. So *"t"he line has no common point with the circle.* This is a very finding. It is not clear if two lines (or other objects) which cross each other have a common point. (Fig. 2)

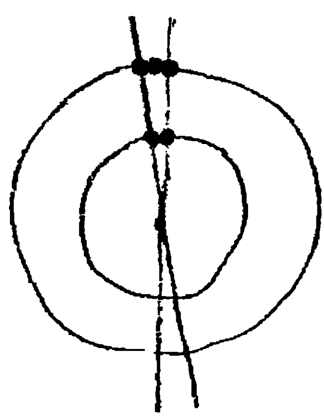

Figure 2.

We get also answers that both segments or circles have the same number of points. The answer of a 12 years old student: "*The same number of points is on both segments. I can assign both endpoints. Then I find the midpoint of the line segments and assign them to themselves. And then I find the next midpoints and make the same thing. And I can make this again and again.*" This student is able to compare cardinalities of the two sets; he is fit to abstract thinking and to understand the idea of limit. Boy's argumentation is potential. We don't know if he realizes a possibility of finishing of this process. (Fig. 3)

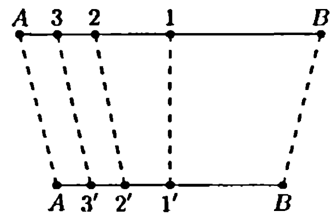

Figure 3.

A questionnaire was carried out with 123 pupils of the secondary school. The main matter of the questionnaire was what is the difference between formulations "How many points belong to ..." and "How many points can be placed on ...".

The questionnaire:

1. How many points belong to the following geometrical objects?
   line, line segment 3 cm long, plane, circle with radius 3 cm, ray, disc with radius 3 cm
2. How many points can you place on the following geometrical objects?
   line, line segment 3 cm long, plane, circle with radius 3 cm, ray, disc with radius 3 cm
3. What is the difference between these questions?

Some results of the questionnaire are presented in the following table.

Line: 1. question	2.question	percentage	percentage
infinity	infinity		27.1%
0	infinity	30.4%	
2	infinity	14.8%	45.2%
0,1,2	some number		13.1%
0	0	2,2%	
1	1	2.1%	
2	2	6.5%	10.8%
other			3.8%

Table 1.

As we can see, 45.2% of the pupils think that the questions are different. Pupils often said that a point is not on a line until it has been marked out. I consider these pupils' theory of the disparity between the two questions as a cause of misunderstanding between a pupil and a teacher in many situations. We'll single out some cogent pupils' answers.

Two pupils said that "*there is an infinity many points on a line, but we can add no one because all points are there and when we add some point we get a new object*". It can link to their actual conception of infinity in this case.

One girl answered that two points are on a line. The answer of the second question was: "*a lot, but countable*". She explained: "*when somebody puts points on the line he can not make from components a continuum*".

Conclusion

We can identify similar phenomena of children's conception of a point and a straight line like in history. Pupils often receive the idea of atomism, which was formulated by Demokritos in 4^{th} century BC and was used by Viète or Kepler in 17^{th} century.

Children in this age don't admit the existence of a segment without its endpoints, similarly like Euklid in his 3^{rd} definition of his *Stocheia*: "Borders of a line are points." (Grammes de perata semeia.)

$45,7\%$ of pupils (from 123) said that a straight line has no point. It gives evidence about their "Hilbert's feeling".

A part of children starts to appreciate a presence od phenomenon of infinity in the smallness of a point (it's more difficult) and number of points on a segment. They can work with potentially infinity processes. But we can't say if they are able to realize infinity in this cases in its actually form.

References

[1] P. Eisenmann, *Fenomén přímky a kružnice ve výuce na ZŠ*, In. Matematika v přípravě učitelov 1. stupňa ZŠ, Bánská Bystrica, 2001.

[2] M. Hejný, a kol., *Teória vyučovania matematiky 2*, SPN, Bratislava 1990.

[3] M. Hejný, *Aj geometria naučila človeka myslieť*, SPN, Bratislava 1990.

[4] M. Hejný, N. Stehlíková, *číselné představy dětí*, PedF UK, Praha 1999.

[5] P. Servít, *Eukleidovy základy (Elementa)*, JČMF, Praha 1907.

[6] P. Vopěnka, *Úhelný kámen evropské vzdělanosti a moci*, Práh, Praha 2000.

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