Annales Academiae Paedagogicae Cracoviensis
Folia 16 Studia Mathematica III (2003)

Jana Příhonská

The "sets of generated problems" with a view to use the graph coding close to solving of given problems

Abstract. The Graph Theory is not a component of standard syllabus of the elementary school, nevertheless some ideas of this theory are used in solving different problems, for example to visualization, systematization of some situations described verbally or done by geometry. The possibility of taking advantage of graph encoding in solving problems of the labyrinth type is discussed in this article. We wiil also gradually proceed to other types of problems. Sets of "generated problems" were compiled based on our own experiences collected during the basic and "pilot" research of using graphs in primary school. Solving these problems contributes to the development of solving strategies of pupils. Our aim was an improvement of the ability to operate with encoded symbol systems in different areas of mathematics at the primary school level.

Sets of problems that arise can be different. The term "**Set of generated problems**" is used by Prof. Kopka in his book; in English texts we can find the terms "**Family (group) of problems**".

The problems in particular sets are configured thematically, according to the aim that should be reached by the pupils. Practicing own situation modeling by using graphs, which for our purpose could be named "graph coding", is the teaching matter. We don't find this term in graph theory, but to a certain extent it reflects our purpose when we convert the accepted problem to a problem of graph theory.

Pupils practice methods of searching a graph, looking for the shortest way, construing of the "Euler line", work with non-orientated graph, systematic solution. Our aim is to make the situation most simple. The possibility of graph coding or (if you like) modeling of a given situation by using graphs is not suggested to the pupils at each of the proposed problems. Pupils learn how to use a graph when they solve labyrinth-like exercises. Especially we speak about so called "labyrinth of color gates". Possible application of graph coding in some problems is shown to the pupils before the setting problems to solve.

We expect then that the pupils will themselves find a graph conception of other problems and they will we use it in their work.

In the next part of this contribution I'll show some problems that can be used as a motivation for pupils in lessons that are suitable for the introduction of graphs at primary school already.

SET 0 — MOTIVATING PROBLEMS

We consider this set motivating. Modeling of a given situation by means of a graph and explanation of our graph coding method for various types of problems is shown to pupils working on problems that are presented here. We don't consider it a classic set as it doesn't include exercises with the same topic and gradation of difficulty, but it is compiled of various types of problems. Types of the problems are exemplified below.

- **"ZEBRA"-type problems** — for lower classes

  These problems are intended for practicing logical thinking. The data presented by the set of exercise can be generally divided into n groups according to relevant data. Then each group includes the same number k of data. The aim is to make n-tuples $(u_1, u_2, \dots u_n)$ so that u_i pertain gradually to i-tuple for $i = 1, \dots, n$. Elements u_i of n-tuples are altogether different. In the view of graph theory we speak about making n-long cycles in a graph of $(n \cdot k)$ points. We don't change conditions given in the problem.

- **Diagram of consecutive activities** — for higher classes

  If we pose a problem verbally, it will be relatively difficult to solve. Using the graph model for a chart with e.g. specification of all activities necessary for the construction of a house and time needed for each activity makes this problem quite easy for the pupils.

- **Looking for the number of routs from start to goal** — for both lower and higher classes

  If we move on a number grid following a given rule the numbers make the same succession as the numbers of branches of the Fibonacci tree: two nearby numbers are always followed by their total. The "strange Fibonacci tree" grows according to the given rule: each side branch starts to grow **up** next year; a **new side** branch grows from each branch that grew up last year. The way of growing you can see in the picture 1.

The number of branches in a single year makes succession "1 2 3 5 ..."; we call that Fibonacci succession. Exercise H0/2 shows this connection of numbers. We can reach a given point through nearest points only, from which arrows go to that point (we keep the permitted direction of the process).

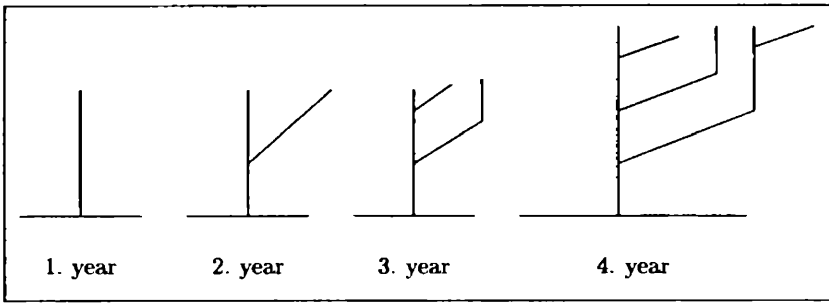


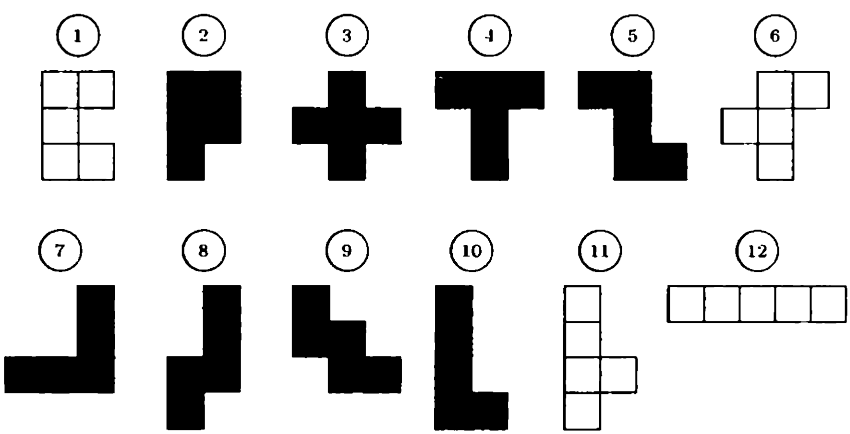
Figure 1. The number of branches of Fibonacci tree

- **Combinatorial problems** — for both lower and higher classes

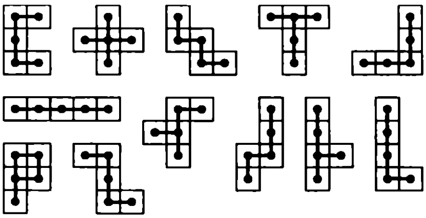
  In the main, the number of all given possibilities according to the problem is to be found (e.g. well known problems about shaking hands, matches in a tournament etc.). Probability problems can be proposed to this group, too.

- **"PENTAMINA"** — according to difficulty, suitable for lower and higher classes

  Pentamino is introduced to pupils along with the possibility of using graphs for solving problems. We introduce the mentioned term graph coding in this kind of problems. PENTAMINO **consist of twelve various cubes, each of them has got 5 squares.**



A certain graph that we call **"pentagraph"** for our purpose corresponds to each pentmino-cube. For better visualization the relevant "pentagraph" is located directly into the given cube (see below).



Cubes Nr. 2, 5, 6, 8, 10 and 11 are the same if we turn them according to vertical axis:

Figure 2. "Pentagraphs"

Text of some problems

H0/1

Various birds have their nests in different types of trees. The nest of each bird is absolutely different from other. The raven nested in a pine, the starling lived in a maple, the crow made its nest of branches, woodpecker had its nest in a cavity. At the top of poplar the nest of branches could be seen, the raven did not nest in a wooden box. Where was a straw nest? Who nested in an oak?

Solving: We will make n-gone, in this case a triangle. We will mark the elements of one group of each side. We joint with line elements that have some connection according to the text of the problem. It is better to use different lines for particular connections.

Answer: The crow has got a nest of branches in a poplar, the woodpecker lives in a cavity of an oak, the starling nests in a box in a maple, the raven has got a straw nest in a pine.

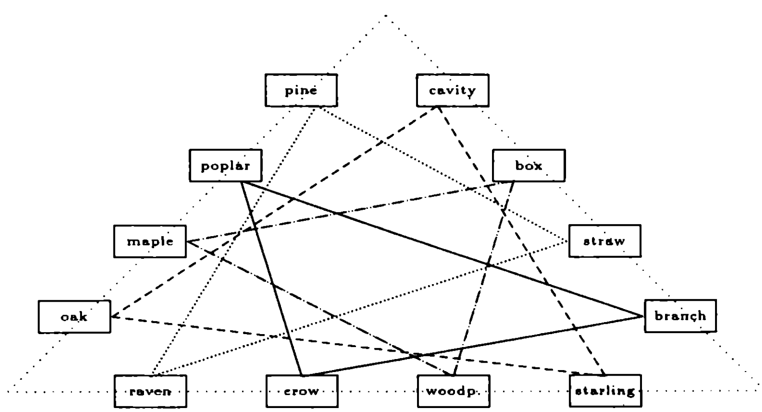


Figure 3. Graph scheme

H0/2

Tourists go up the hill. There are some bending roads, the path with turns, to the right, to the left, to the right, etc. (see Fig. 4). At places where the turns bend you can rise even straight. If you do not mind steep rising you can sometimes shorten the route.

Task: Write the number of ways that the tourists can use to each point where the ways branch. They will never go back, of course, they will always rise along the arrows. Number 1 is at the start as there was no crossroad before that and you could get there only one way.

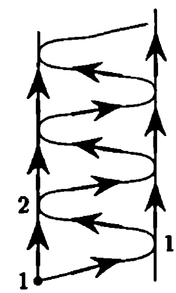


Figure 4. Scheme of a plan

We can use a simple draft if we want to find the number relevant to each of the crossroads. We can get to the next point only through the foregoing point and the number of possibilities is the total of both foregoing points together. The graph in picture 5 always expresses the number of possible routes that we can use to get to a certain place.

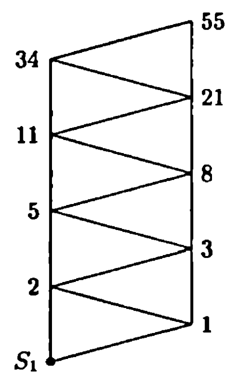


Figure 5. Repaint of the plan with solving

Answer: There are 55 various possibilities to reach the top of the hill.

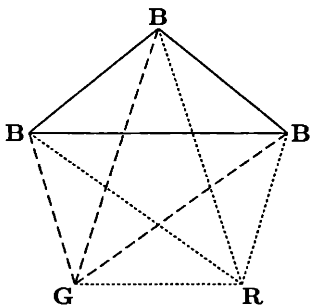
H0/3

We put 3 blue (undistinguishable) balls, a green one and a red one into a box. We mix them up and take two balls out (we do not put them back and do not look at them).

Task 1: Is it more probable to take out two balls of the same color or of different colors?

Task 2: How many possibilities for the choice of two balls are there?

Solution: For better visualization we will use a graphic description of possible choices of the two balls. In the next picture choices of two balls of given colors are always linked by the same type of line, the enumeration of all possibilities is recorded beside the picture.



B – blue ball
G – green ball
B – red ball

BB – 3 possibilities
BR – 3 possibilities
BG – 3 possibilities
GR – 1 possibility

(BR, BG, GR: 7 possibilities)

Figure 6. Graphic illustration of the choice of the balls

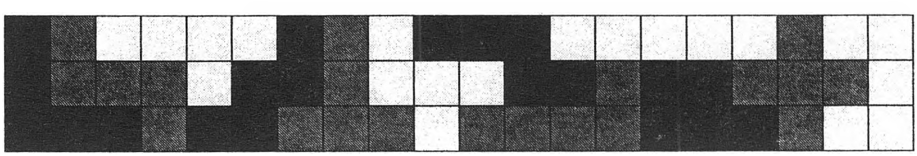
The calculation of all possibilities of the choice of two balls:
Each of five balls we can combine with any of the left four ones (the order of taking the balls out is not important): $(5 \times 4)/2 = 10$.

Answer: The choice of 2 balls of different colors is more probable.
The total of different choices is 10.

H0/4

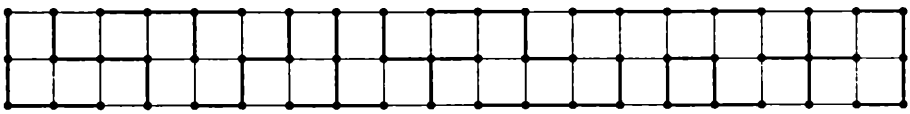
We will take an oblong 3×20 on square network. Our task is to fill the oblong with twelve various pentamino-cubes.

Solution:



Let us use a graph for this problem:

Task: Place all 12 *pentagraphs* in the square network 2×20. No two *pentagraphs* may share a point, all points of the network must pertain to some *pentagraphs.*



H0/5

How many possible ways go from S̲chool in the left upper corner to the H̲ouse in the left lower corner (you can move along the arrows only)?

Solution: See Fig. 7a, for better visualization we use the h-diagram (see Fig. 7b).

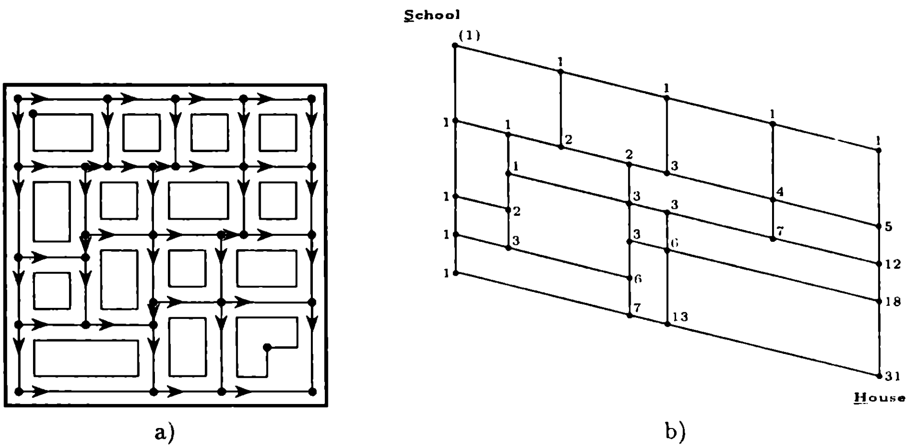


Figure 7. Plan of housing estate and h-diagram

References

[1] M. Hejný, *Barevná bludiště*, PedF UK, Praha 1998, SPN Bratislava, 1990.

[2] J. Kopka, *Hrozny problémů ve školské matematice*, Ústí nad Labem, 1999.

[3] *Precalculus and Discrete Mathematics*, The University of Chicago School Mathematics Project, Scott, Foresman and Company, Glanview, Illinois, 1992.

[4] J. Příhonská, *Teorie grafů v učivu základní Školy*, In: Sborník Mezinárodní věd. konf. "Matematika v príprave učitel'ov 1.st. ZŠ", Univerzita Mateja Bela Banská Bystrica, 2001, 40-46.

[5] J. Příhonská, *Řešitelské strategie s využitím teorie grafů v učivu základní Školy*, In: Sborník mezin. věd. konf. "Podíl matematiky na přípravě učitele primární školy", Univerzita Palackého v Olomouci, 2002, 149-153.

[6] J. Příhonská, J. Vild, *Hasseovské diagramy ve výuce, In: Sborník mezinárodní konference kateder matem. připravující učitele matematiky*, TU v Liberci, září 2000, 91-96.

[7] S. Vejmola, *Konec záhady hlavolamů*, SPN Praha, 1986.

Technical University in Liberec
Faculty of Education, KMD
Hálkova 6
461 17 Liberec
Czech Republic
E-mail: jana.prihonska@vslib.cz