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Perception of rectangular and skew structures in a row-column arrangement of figures

Abstract. The aim of the reported research is identification of difficulties and natural developmental limitations in understanding of a Row-Column Arrangement of Figures (RCAF) by 10-13 years old children. RCAF includes the arrangements made by regular distribution of any elements in rows and columns. These elements don't have to be the same and adjoining.

Analyzing school curricula shows that the multilateral understanding of RCAF is a prerequisite knowledge for many problems; e.g. the product of two positive integers, the commutative law of multiplication, the idea of area, coordinates, matrices and various tables.

In their tasks children had to construct and draw examples of RCAF. My research results show which elements of the given structure are important for children and which of them are dominant in the children's awareness. The research also proves that at the beginning of school, children do not possess well formed structures, necessary for the understanding of some mathematical ideas or operations, while the curriculum assumes their presence. The introduction of concepts based on unformed structures may lead to dramatic failures of some students.

Introduction

The research¹ presented in this paper concerns the understanding of a certain type of regular arrangement of figures in the plane, which is called a row-column arrangement of figures (hence the abbreviation RCAF) as manifested by pupils aged 10-12. The essence of a RCAF is a possibility of distinguishing — within its frame — two families of parallel rows intersecting at a certain angle. The figures we refer to are situated at the points of intersection of respective rows. The elements of a RCAF may be arbitrary figures, not necessarily congruent to one another; what is important is their position. For example, the position of figures may be defined by a two-dimensional net of points with integer coordinates in an oblique system of coordinates in the plane.

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The studies performed so far² which dealt with the understanding of this class of arrangements allowed to discriminate three types of structures which attract children's attention: set type of structures (characterized by families of sets created of the figures forming a particular arrangement), numerical type of structures (related to numerical relations between the discriminated sets of figures), surface type of structures (concerning relations between the surfaces taken up by the figures of a particular arrangement).

The research described here focuses on two types of the set type of structures: a rectangular structure and a skew structure. Figure 1 shows the net of points — an example of an infinite RCAF, in which a rectangular structure and several skew ones have been schematically distinguished. We are dealing with a rectangular row-column structure if within the arrangement a family of parallel rows and a family of parallel columns may be discriminated and the whole arrangement is treated as two families of perpendicular rows. If these two families of parallel rows are not perpendicular to each other, then we will say that it is a skew-rows structure. The core of our research was to ascertain if children are able to notice both structures in the same RCAF.

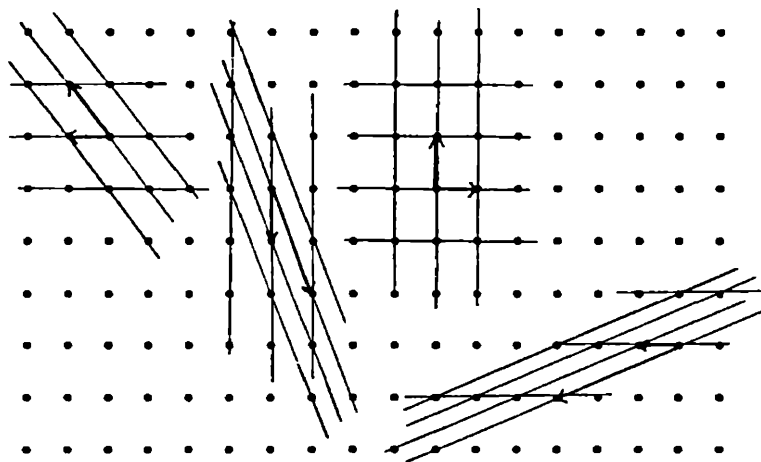


Figure 1.

The structure and objective of the research

The research was conducted in 1999 among pupils from V-VII grades (children aged 10-13). The total of 30 pupils participated in the study. The main research method employed was the diagnostic-problems method accompanied

²Between 1992 and 1996 the research on the understanding of the RCAF by children aged 6-9 was conducted. Its results were published ([5], [7]). The comprehensive description of the research was presented in my doctoral thesis ([8]).

by observation and an analysis of pupils' output. Each pupil was working separately in the presence of the researcher. The objective of the study was to answer the following questions: What types of structures do children notice in the RCAF? Are they able to notice two different structures in the same RCAF?

In the course of the research pupils were presented with two problems:

Problem 1. *The chart shows the pattern of some fabric reduced in size (Figure 2). These are two tablecloths (Figure 3 and Figure 4). Were both these tablecloths cut out of the fabric of such a pattern? Why do you think so? Can you convince me that you are right?*

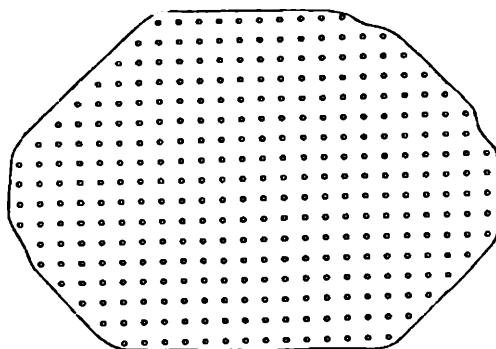


Figure 2.

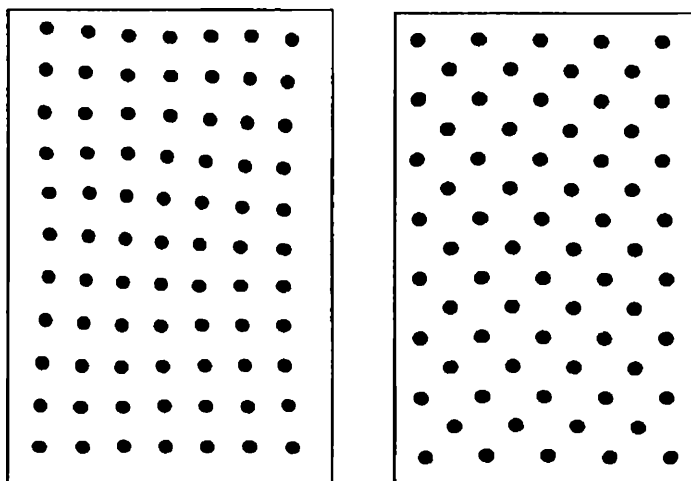


Figure 3.

On the paper chart the pattern of the fabric was represented only schematically as black circles creating a row-column arrangement of figures. The

tablecloths were made of a white fabric with red dots. The distances between the dots on the paper cloth were different than the ones on the tablecloths. In the models of the tablecloths two structures were emphasized: a rectangular one, sized 7 by 11 dots (fig. 3), and a skew one, 5 by 8 border dots (fig. 4). The aim was to investigate if children are aware of the existence of these two structures in the same RCAF. Regardless of the answer given, a pupil was presented with the second instruction.

Problem 2. *Cut out the same tablecloths of our paper cloth. Our paper cloth was reduced in size, so the tablecloths you cut out of it will also be smaller.* After the children have cut out the tablecloths we show them the previously prepared tablecloths cut out of the paper fabric and we ask them:

Do the rectangles I have cut out represent our tablecloths?

The models of tablecloths correctly cut out were a necessary aid to the next problem. Its goal was to attempt, in case of failure to solve the second problem, to awake the awareness of the fact that such two kinds of tablecloths may be cut out of this fabric.

Four types of pupils' behaviour

The analysis of the first and the second problem allowed to distinguish several types of behavior of the participants of the study. Each type will be characterized by examples of pupils' solutions.

Type 1: NO-NO

The classification NO of this type of behaviour means that in the first problem a pupil stated that it was not possible to cut out both tablecloth of the fabric of the same pattern. The repeated classification NO means that a pupil did not succeed in cutting out the skew tablecloth.

The pupils notice different structures in the tablecloths they were presented with, nonetheless, the most frequent statement here is that only the rectangular tablecloth may be cut out of the given fabric. This observation is backed up by the following remarks of the participants:

- *The other tablecloth* (about the skew one) is not made of this material, these are straight and these are at a slant (Pawel, aged 13 years 11 months);
- *These are arranged side by side* (indicating a square on the rectangular tablecloth) and here (about the skew one) *a circle is missing, here there are squares with a circle in the centre* (about the skew one) (Kasia, aged 13 years 9 months).

Some pupils from this group gave the negative answer to the following question: Have both these tablecloths been cut out of the fabric of such a pattern? because they interpret it as follows: Have both these tablecloths been cut out of this fabric? Here are some examples of remarks made by these pupils:

— *No, because the dots on the tablecloths are bigger and they are red* (Piotr, aged 12 years 5 months);

— *I can see no relation between the fabric tablecloths and the paper pattern* (Katarzyna, aged 13).

During the process of cutting out some pupils are trying to produce the skew tablecloth but as a result of this action they obtain a rectangular one.

Type 2: YES-NO

The classification YES means that a pupil stated that both tablecloths were cut out of the fabric of the same pattern, whereas the classification NO indicates the fact that in the second problem a pupil did not succeed in cutting out the skew tablecloth. The most frequent reaction of the pupils of this group is noticing that to create the skew tablecloth it suffices to rotate the pattern. Nevertheless, in the course of active working on the problem they fail to produce the skew tablecloth. Answering the question: *Have both these tablecloths been cut out of the fabric of such a pattern?* Piotr (aged 13 years 9 months) says: *Yes, two of them. It's of the same pattern... Sort of squares made of circles.* Referring to the skew tablecloth he claims: *This one too, because here we also have squares, only they are placed at a different angle. If I tilt this fabric it turns into this tablecloth.* For Piotr the number of dots is a significant factor. He counts the elements of the rectangular tablecloth and cuts out the same arrangement of 7 by 11 dots. Next, he counts the border dots on the skew tablecloth, counts 5 by 8 dots on the fabric without tilting it. However, he notices that in this way he will not obtain the skew tablecloth, so he says: *I don't know how to cut it out of this thing... I suppose this one (about the skew tablecloth) is not made of it because there's a circle in this square and there's no circle on the fabric.* Piotr has changed his initial belief and now he is convinced that the skew tablecloth has not been obtained from the same fabric as the rectangular one.

Type 3: NO-YES

The classification NO means that a pupil stated that both tablecloths were not cut out of the fabric of the same pattern, whereas the classification YES indicates the fact that in problem 2 they succeed in producing both tablecloths. In the course of discussion the pupils most often claim that it is not possible to create the skew tablecloth. However, during an attempt of solving the problem they succeed in positioning the fabric in such a way that the skew tablecloth is formed. Some pupils create tablecloths of the skew pattern without preserving the number of dots, others pay attention to the number of elements as well. A success in creating the skew tablecloth was sometimes the result of the fact that a certain conflict emerged, like the one that appeared during the activity of Krzysiek (aged 11), who says, referring to problem 1 and the pattern of the tablecloth: *This one is different (about the skew one), the arrangement*

is different, these are not squares but rhombuses. The number of elements is important to him, so he counts the border elements on the tablecloths. First, he cuts out the rectangular tablecloth, preserving the number of the dots. Then he counts the border dots on the skew tablecloth, 5 by 8, and he counts the same number of elements on the fabric but he imposes the rectangular structure on his arrangement. The boy scrutinizes the tablecloth he obtained and says: *Definitely not.* The conflict that emerged induces him to undertake another attempt of producing the skew tablecloth. This time he turns the fabric and counts 5 by 8 dots on the skew rows obtaining the proper skew tablecloth.

Type 4: YES-YES

This type encompasses the following pupils' behavior: YES, it is possible to cut out both tablecloths, and YES, they succeed in producing both tablecloths. The pupils from this group notice immediately or after a short while that both tablecloths may be obtained from the same fabric. They also succeed in producing both tablecloths in the course of the practical part of the activity. Przemek (aged 13 years 3 months) claims: *Yes, the skew pattern is only tilted, it is the part of the fabric. The second one is also made of this fabric, the dots are vertical and horizontal* and he cuts out both tablecloths preserving the number of dots. Some pupils, however, pay attention just to the pattern of the tablecloth and do not preserve the number of elements.

Summary

In relation to the objective of the research defined in the beginning it may be stated that the children who took part in the study were able to notice different structures in a RCAF, both the rectangular structure and the skew one; they even observed the differences between these structures. Nevertheless, the majority of the participants were not aware of the fact that these two structures existed within one arrangement, that they could be identified in the same RCAF. The shift from perceiving one structure to noticing the other one caused a considerable amount of difficulties. Grasping the skew structure in a pattern was much more difficult for children than representing the arrangement on the basis of the rectangular structure. What considerably influenced the type of the structures perceived was the way in which a particular RCAF was positioned with respect to the child working with it. The structure which dominated was the one in which the rows were horizontal from the point of view of the observer or with respect to other elements, like, for example, the edge of the chart, and it is possible that this was the factor that caused this structure to become the dominating one and to block the perception of other structures.

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