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Combinatorics' A journey to elementary mathematics education

Abstract. Combinatorics is a part of discrete mathematics. It is a part of modern mathematics that seem to penetrate into the mathematics subject taught in school. Combinatorics forms an inseparable part of the modernization of elementary mathematics education. In the paper we investigate the options of structural integration of combinatorics into math education at the elementary school level and the combinatorics-pupil and combinatorics-math teacher relationships.

Introduction

"Combinatoric concepts can be traced in some riddles, arithmetic and geometric problems' solutions generated by ancient civilisations (Greece, China). However, it was only in the latter- day mathematics that combinatorics evolved into a mature discipline, mainly due to theoretical works of Euler, Laplace, Pascal and Fermat. Its modern foundations are intertwined with the essentials of the graph theory. Combinatorics currently represents a developing part of discrete mathematics" ([6], p 288).

The present times, sometimes characterized as the information age, pose totally new demands on mathematics. 'Continuous' mathematics, built on the real axis and geometric visualizations and objects, is retreating from its dominance and ever growing importance is attributed to discrete mathematics (with combinatorics as its integral part), probability theory, statistics, numerical mathematics, logic, and number theory.

Adjustments in the syllabus and making it more attractive in primary and secondary schools are some of the principle goals for all those strivings for a positive image of mathematics throughout the society. This requires implementing a structural shift from traditional school-mathematical themes to other areas of mathematics, mainly discrete mathematics (including combinatorics), informatics, probability theory, and statistics. Attractiveness of these areas is in the fact that their applicability in real life can successfully be demonstrated and validated even in elementary grades (according [4]).

Combinatorics and Teaching Mathematics

When a suitable methodological strategy is chosen by the teacher, the elementary combinatorics exercises, such as determining the number of selected objects in a few-element finite group, ordering and tallying them, are certainly not too difficult to be listed in school mathematics syllabi. Pupils can familiarize with combinatorial principles via games and solving problems.

Why is it useful to integrate combinatorics into a mathematics syllabus? At first, it is the attractiveness of combinatorics. Pupils can find many problem situations interesting, thus providing them possibilities of investigating and disclosing. At second, there is a wide range of activities suitable for excellent pupils as well as for those less successful in mathematics. At third, it is its approachability. Arithmetic and elementary algebra are sufficient to understand the principles of many applications.

Elements of combinatorics appeared in math syllabus long ago. Whenever the selected objects are counted, ordered and arranged, combinatorics is introduced.

According to Balint [1], it is useful to advance in teaching combinatorics as follows:

1. Pupils search at first one, later several solutions, e.g. number, word or way. This enables the teacher to find out whether they understood the conditions and know what to search for.
2. They try to find as many various solutions of the task as they can.
3. They search all the possible solutions. It should be clear that they have found all the solutions. It is achievable when they find a certain order in the tallying process.
4. Pupils need not know all solutions, only an "order in them" based on how they surmise the continuation and the number of solutions.
5. It is not necessary for pupils to enumerate all cases, since based on the analyses they can deduce the number of all solutions.

Hejny ([2], p 472) emphasizes: "Combinatorial thinking is built up on the ability to organize elements of a set to clearly arranged tables, graphs, schemes and lists." The notion combinatorial thinking implies abilities to create an abstract model and find arranging principle; the center of pupils' combinatorial abilities is in their readiness to find the arranging principle.

The way of teaching combinatorics remains an important question. According to Hejny/Michalcova [3] combinatorics is the part of a syllabus in which processional approach is clearly distinguishable from the conceptual one. The processional approach starts with an insight into combinatorial situation and continues with further arrangements. Conceptual approach requires an understanding of the existing formula for particular type of combinatorial group. In

the past, when combinatorics was taught within an optional subject Exercises in Mathematics or in classes with extended study of mathemetics, the conceptual approach prevailed. It is evidenced in the textbooks from this era. They proved that combinatorics in written form is presented rather more easily in the conceptual than the processional way. However, combinatorics offered in this way was difficult to understand. All-round unpopularity among pupils is demonstrated by this fact. Most pupils view combinatorial situations as processes. The viability of processional approach in the teaching of combinatorics at elementary school is proved by some partial research results as stated in [11], [12], [13], [14].

We have conducted several didactic experiments focused on developing combinatorial thinking among elementary school pupils. The principal idea behind it was that combinatorics is not a closed area of mathematics that must be presented in one thematic syllabus. Its elements occur and can be incorporated into most themes in mathematical syllabus. Combinatorial tasks built up on the thematic context of the syllabus taught at a particular stage were presented to pupils in the course of the school year. They were not informed that what they were solving is combinatorics. Based on the first analyses it was evident that those pupils apply processional approach. They analyzed combinatorial situation and consequently arranged them. If the combinatorial situation was understood correctly and appropriate organizing principle was chosen, they were successful. The detailed quantitative and qualitative analysis of the didactic experiment is to be made in near future. Based on its outcomes we can submit data on developing combinatorial thinking in the teaching of mathematics at elementary school to our colleagues for evaluation and inspiration.

Combinatorics and the pupil

The perception of combinatorics among pupils, especially at the secondary level, is best illustrated by one student majoring in mathematics and physics: "Combinatorics is like a lottery. I can never decide upon a formula for combinations, variations or permutations. It is usually a guess. I lack a solid soil underneath, therefore I dislike combinatorics" (quoted in [2], p 472).

A student who encountered combinatorics at the secondary school, where it was taught conceptually, holds this view. Combinatorics presented as a set of certain formulae that should "fit" in to the task's wording would not be encouraging.

Since most pupils treat the combinatorial situations in processional way, such approach of teaching it seems to be more useful.

There are many combinatorial situations and problems (tasks) that can attract not only school age children (6–8 years), but also 3–5 years old.

Familiarization with combinatorics can start with manipulative activities of a child. What tower can be constructed from colored cubes, what flags can

be assembled from color stripes, what bouquets cans be assembled from various sorts and colors of flowers, how many ways can we dress the doll combining several blouses with skirts etc.

In the next stage the pupil should become aware of the necessity to organize work, thus creating a system in every activity. Combinatorics serves this purpose utmost. To combine is the meaning of the verb in everyday activity. Starting from early morning we all combine: what to have for breakfast, in which turn shall we go to bathroom, what to wear, what is the best combination of lanes in public transport to get to school fastest, in what sequence we perform our after-class activities, what and for how much we buy while shopping, what to prepare for dinner, which TV program on which channel we are going to watch, etc.

Pupils must be aware of the fact that everything around is ruled by certain systems. Once the system is impaired, our life becomes more complicated.

Rosenstein [7] formulated essential skills in discrete mathematics pupil may posses in a particular age. For combinatorics the following facts result.
Fifteen years old pupil ought to:

- Know how to arrange a set of all possible cases of a single situation (what are the options in deciding what to put on, provided there are three hats and two coats);
- Know that a layout of streets can be represented by graphs and the streets correspond to the edges of the graph;
- Understand that multiplication is repeated adding of the same number;
- See that pattern on the decorated floor is formed by repetition of smaller size pattern, rows in a pinecone are the result of a simple mathematical principle;
- Be able to order information in tables, graph trees or diagrams;
- Be able to act according to instructions in order to get from one place to another. Fifteen years old pupil ought to:
- Know to enter in variables systematically and determine the number of selected objects of a finite group;
- Know to allocate all the possible routes on the map leading from one place to another;
- Know to compile the most favorable routes with respect to price connecting the locations into a network by means of a branching tree;
- Know how to explore simple repeated patterns (mosaics) and create them;
- Be able to read, construct and analyze tables, matrices, maps and other data structures;
- Be able to plan the most suitable rout for a group trip;

- Know to describe exact instructions for adding two digit numbers.

Combinatorics opens for students new possibilities. The traditional themes in school mathematics — arithmetic, algebra, geometry etc. — are important as well and it is necessary to have a solid basic knowledge about them. There are, though, many pupils constantly failing in mathematics. For them, the subject is just a set of incomprehensible procedures. They have never had a chance to explore processes bearing practical meaning, thus applicable in real life situations. The second group includes gifted pupils for whom school mathematics is unattractive and neglected. They focus on other areas. Combinatorics is therefor offering a new start. For those constantly failing in mathematics, it is offering a chance to be successful. They are further encouraged taking a different view on math. They find out that they can solve more complex problems and have a sense of 'grasping' mathematics. For talented pupils, who lost their interest in the subject, combinatorics is offering a domain of more difficult or advanced tasks, open ended problems leading to the limits of knowledge.

Combinatorics and the Mathematics Teacher

Most of currently active mathematics teachers have undergone a conceptual way of training in combinatorics. Not surprisingly, they are 'set on' conceptual teaching of combinatorics in their school practice. Since this part of mathematics was not favored by many of them, their aversion to the subject, hidden in many cases, is transferred unconsciously on their pupils.

According to the questionnaire inquiry on teaching combinatorics surveyed among mathematics teachers (reported in [8] and [9]), interesting outcomes were received.

- For most of the teachers, combinatorics is a part of mathematics, in teaching of which they do not have enough experience (in Slovakia combinatorics entered in the curriculum only in 1997).
- There are instances of insufficient combinatorics knowledge.
- Teachers feel the need to upgrade their expertise especially in the methodology of teaching combinatorics.
- Listing combinatorics in the curriculum is considered as suitable.
- Many teachers do not have a clear idea on what should be taught in combinatorics. Consequently, there are difficulties in setting out educational goals, which points at insufficient methodological training of this topic.
- Personal feelings about teaching combinatorics are more or less neutral.
- If there is a choice between 'traditional' and unconventional mathematics textbook, the former seems to be the preferred in most cases. Reluctance to change is deeply rooted.

A weak point in the teaching of mathematics is in applying mechanical approach to education. It is often forced by public demand for successful results at the entry exams to secondary or tertiary level. Teachers try to program pupils by means of drill to perform arithmetic, algebraic and geometric operations so that they be able to solve a task that can be classified and solved according to adopted pattern. Such an approach is hardly utilized in solving combinatorial problems. 'Away' from combinatorial tasks we are 'navigated' by the designers of mathematical secondary school entry tests. Based on the tests' analysis (reported in [10]) it appears that despite the fact that combinatorics is an integral part of basic school curriculum, combinatorial tasks rarely occur at the entry tests. (Similarly, probability and statistics never appeared at the entry tests!) Their proportion in the four-year-school tests is clearly insufficient The situation with entry testing in eight-year school is slightly better; combinatorial tasks were found in more than a half of the analyzed test samples.

In spite of the above, it is promising that there are many elementary school teachers who implement a creative approach in the teaching of mathematics. We have been assured about this during pedagogical experiments with teaching combinatorics and during workshops organized within further education.

Conclusion

Hence combinatorics, so unpopular and evasive, is trying to find its way into elementary mathematical education. However, its entry can not be a violent one. According to Płocki ([5]), one of the key goals of mathematical education is students' getting accustomed to the methodology of mathematics. It includes their grasping of mathematical processes , organizing pupils' mathematics activities and developing intellectual attitudes typical for such activity. So is the principle goal of combinatorics, to familiarize with the methods and ways of thinking that are characteristic for combinatorics. Its aim cannot be reduced to transmission facts and formal knowledge to the pupil. The main goal therefore should include formation of the cognitive side of pupil's personality, i.e. developing his/her abilities to think respectfully for combinatorics' principles.

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