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Space imagination as a precondition for geometry learning

Abstract. In the contribution, some experience acquired in the university education of primary school prospective teachers of mathematics will be presented. Space imagination is the ability needed by everyone. Prospective teachers should know activities, which develop space imagination of pupils and should use them in their teaching.

1. Introduction

Geometry can be seen, firstly, as a way of understanding and exploring the world, secondly, as a covert sense of different schools of thought, thirdly, as a way of "storing" information, and finally, as a part of mathematics having clear connections to everyday practice (e.g. techniques, arts). Thus, through geometry lessons, a learner gains means for solving problems in the real life and in most professions and develops a very important skill — space imagination.

As it is often stated in the teaching of elementary geometry, both future primary school teachers and some future primary and secondary mathematics teachers (teaching the 5^{th}–12^{th} grade) possess a weak space imagination ([1], [2]). According to a research aimed at gathering students' opinions on geometry and space imagination in geometry lessons at primary and secondary schools, 70% of future teachers fully understand the importance of space imagination for geometry problem solving. However, only one quarter of the respondents are convinced that their own imagination is fully developed ([4]). The problems with developing the notions of geometrical figures and space imagination were also encountered during observations at primary and secondary schools. The teachers who themselves have space imagination difficulties are likely not to include the tasks involving space imagination in their lessons and to devote enough time and effort to them. A possible reason for this might be the lack of their own experience with such tasks or underestimating the learners' potential.

Space imagination might be defined as the ability to imagine the qualities of three-dimensional objects, i.e. their shape (body form), position, size and placement in the space (direction and distance). All these characteristics can be expressed as a set of relations. The shape determines the relations among the

parts of an object, the size relates to a certain unit of measurement, the position and placement outline the object's relation to its surroundings. Space imagination is initiated and then perfected through a process of observation. Space imagination thus acts as a link between perception and thinking. Geometry involves a constant creation of notions and conceptions of terms. Learners usually do not have difficulties in acquiring exact notions of plane figures. What seems to be more difficult is the understanding of three-dimensional objects.

Space imagination is based on recognizing the shapes of objects, their location and movement in the space. The development of space imagination is closely connected to the understanding and creation of the conception of the term "geometrical figure". Spatial characteristics of objects – shape, size, position and location – are relevant and significant features of two and three-dimensional figures.

An effective educational process is built on learners' experience. During the learning process a learner seeks answers to given questions and creates notions basic for developing the conceptions of terms and pieces of knowledge. Experience, notions and knowledge are combined together and allow the creation of other experiences, notions and pieces of knowledge.

2. Manipulatory activities

Primary school pupils and future primary teachers were tested on geometrical knowledge and skills and were asked to express their attitude to this school subject. It was discovered that the outcomes of geometry lessons are not satisfactory. As a result of that, our attention was paid to manipulatory activities during which a child handles concrete objects and so encounters different geometrical configurations. While handling the objects, touch, a very important sense, is involved. Children discover the characteristics of the objects regardless their position. Manipulatory activities can be seen as multisensory tools supporting learning through learners' own experience, gained not only by sight but mainly by touch.

One of the teaching aids to develop space imagination can be made by pupils themselves. It requires manipulation with three-dimensional objects – cubes – that are to be placed into a box.

There are two basic tasks to solve:

a) to gather the cubes in the box according to the given cards (i.e. — to place the cube in the box if its conjugate projections are given);

b) to draw the situation in the box using orthogonal transformation onto three projection planes (using recording cards).

Learners gradually get to know how to use the aid; the focus should be on raising the difficulty of the tasks and the teaching principles should be followed. At the beginning it is essential to introduce pupils to the rules of the

basic three-dimensional bodies' transformation — one cube — onto one, two or three projection planes. Pupils gradually receive a more accurate insight into the possible uses of this transformation as they continue manipulating with the aid. After the mentioned tasks, pupils can solve tasks based on incomplete cards (e.g. for the exact placement of a cube in the space a sketch of a front-view and a side-view plane is sufficient to know, i.e. two conjugate projections define the third projection).

In the next step, pupils stop manipulating the aid. They draw the ground plan directly according to the conjugate projections (front view and side view) in an incomplete card. The answers can be checked through modelling or on the basis of imagining the situation in the box. While imagining the box set, learners realize the number and colour of the cubes, their relative position and placement in the box.

The teaching aid can be used with slight modifications both with primary school children and university students. In agreement with this, a group of future primary school teachers was instructed about the use of the aid. After a brief introduction of the problem, test tasks were set:

1. To fill a missing ground plan into an incomplete card;
2. To draw the situation using free parallel transformation according to a card with three orthogonal projections.

Ad 1) The tasks with incomplete cards did not cause any difficulties to the students. The problems were solved correctly by 90% of the students. Most errors occurred in the cases where the cubes were situated in more than one layer. Where two or three layers appeared, there were many corrections.

Ad 2) Much more errors than in the previous case occurred when the students were supposed to project the cubes using free parallel transformation according to a complete card.

The task 42A1 (a set with 4 cubes, 2 layers and 1 color) was fully solved by 44%, partially by 46% and incorrectly by 10% of the students. The partial solutions include those, in which one of the following features was wrong — shape, size (related to a square coordinate grid), placement or visibility. The size was determined wrongly by 50% of the students. The solutions containing more than one incorrect feature were included among the incorrect answers.

The task 52ABC2 (a set with 5 cubes, 2 layers and 3 colors) was solved correctly by 42%, partially by 30% and incorrectly by 28% of the students. As partial solutions were considered those with one wrongly indicated feature - shape, size, placement, visibility or color.

From the data mentioned so far, it is apparent that together with the increasing number of colors, there is also a significant rise in the number of errors. Less than 50% of the students were successful in the test, which suggests that students' — future teachers' — spatial imagination is not fully developed.

To be able to teach properly, teachers must be familiar with the correct notions of geometrical figures they teach, as well as with other aspects of space imagination, such as:

- to be able to imagine composite figures as a unification of basic figures;
- to do the in-mind analysis of figures (to estimate and specify the relative position of the subsets of the points creating the figure and estimate and describe the relations among them);
- to create notions of size of the basic units, to estimate the size of figures;
- to describe the situation with the help of geometry terminology and symbols;
- to be able to decide about the space arrangement of geometrical figures (visibility in the graphical representation of the situations in space);
- to develop drawing skills etc.

3. Conclusion

In the end, several recommendations for exploiting the outcomes in specific school situations should be mentioned.

The manipulation should be included almost into all class activities. Teachers can use specially designed teaching aids, available on the market (various construction sets), or aids produced by learners themselves may sometimes be sufficient.

It is considered beneficial for teaching mathematics to create a system of tasks for the continual and meaningful development of space imagination.

In addition to that, the interdisciplinary approach should be employed to search for appropriate components to develop space imagination; then, to use the components in arts and technical education, physical education and other school subjects and to coordinate the formative effect of this strategy on learners and their own activities.

Finally, the constructivist approaches to teaching geometry at all school types and levels should be used, the activity-based character of teaching geometry should be emphasized in teacher training and effective means for the development of space imagination as a generally useful skill should be sought.

References

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