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Selected aims of teaching mathematics and the problem-solving process

Abstract. The contribution will present a study, which was meant as an attempt to diagnose the realization of those aims of teaching mathematics that assume modeling active attitudes and behaviors of students during the process of solving mathematical problems. Therefore, the analysis of the problem-solving process became here a specific tool applied to characterizing and describing "a 14-15-year-old student's attitude towards mathematical problems".

The problem of formulating aims of teaching and a subsequent verification of the degree, in which they have been put into practice, is and has always been an important question for both didactics and the practical side of teaching mathematics. It is worth noticing that whereas the process of postulating these aims may be performed in a certain isolation from practical teaching and from students, for estimation of the degree of their implementation such an isolation is unthinkable.

It is a well-known fact that the didactical literature contains numerous typologies an hierarchies of aims of mathematics teaching. The majority of them assume that mere addressing particular topics included in the school curriculum are not the only teacher's task. The objective of equal, if not greater, importance consists in developing in students certain talents, tendencies and behaviors, which may be transferred to other areas of their lives.

The research presented here is an attempt to estimate the results of realization of selected aims of teaching mathematics, which are of a level higher than mere practicing "basic mathematical knowledge and skills" ([6], p. 25). The research objective I chose had been described by Z. Krygowska as concerning "attitudes and behaviors characteristic of mathematical activity (...), for example, an active attitude toward mathematical problems" ([6], p. 26). In my research I assumed that I would be "looking into" the results of realizing the above mentioned aims by an observation and description of students' *attitudes* which may be recognized in the process of problem-solving.

In the course of theoretical studies I realized that in psychology there are several radically different definitions of the term *attitude*. For that reason in

my research I decided not to stick to any of them. In order to direct my studies I assumed that an *attitude* is a three-dimensional structure which comprises three components: behavioral, cognitive and emotional-motivational ([9], [10]). Such an interpretation of the phenomenon of *an approach* helped me to realize what psychologists pay attention to when speaking about an attitude and what I myself, as a researcher, should try to perceive while subjecting students to an examination.

As a result, I formulated the initial assumption of the whole study, namely that an indirect tool for estimation of the realization of these selected aims I would employ the analysis *of the process of problem-solving*, which will allow me, in terms of the psychological language of *attitudes*, to make a preliminary diagnosis of the realization of the aims of teaching, which were selected for this research.

Taking into account the above quoted description of the phenomenon of *an attitude* as consisting of three components (the behavioral, the cognitive and the emotional-motivational one) I split the process of formulating conclusions resulting from my studies into two phases. The first phase concerned the cognitive-behavioral aspect of a *student's attitude toward mathematical problems* while the other one was related to its emotional-motivational component. I conducted a clinical[1] study on a restricted group of students aged 14-15; during several meetings I was observing their work on approximately 20 various problems which might be classified as non-standard[2]. Subsequently, I was analyzing the detailed reports of their proceeding, at first individually and then (when I distinguished certain phenomena which, in my interpretation, might be a source of information on the type of *an attitude toward problems* a student displays) they underwent a comparative analysis. As a result of this comparative analysis I managed to construct a certain typology of students' cognitive behaviors. The whole of the analyses concerning the cognitive-behavioral component of the phenomenon I investigated will not, however, be the subject of the present paper. I will focus on the module of my research which aimed at formulating **a diagnosis concerning the emotional-motivational aspect of students' attitudes toward mathematical problems.**

[1]The research perspective I assumed consists in treating this study (at its first level) mainly as a case study in the medical meaning of this term. This medical metaphor is quite convenient in this context. It reflects the fact that information on one student are obtained from various sources, similarly to the results of medical tests.

[2]M. Legutko ([7]) assembling various attributes of non-standard problems distinguishes three aspects according to which a problem may be classified as either a standard or a non-standard one, namely: "a problem may be standard with respect to a) the mathematical contents, if it is directly connected with knowledge and skills required by the curriculum, b) schematic solution, if a student may solve a given problem by direct application of a familiar pattern, c) the problem's structure, if it contains the exact amount of data that is necessary for solving it, the instruction or the question is clearly stated, the problem has the unique solution (result)" ([7], p. 16). A problem, therefore, may be classified as a non-standard one if it does not meet one or more of these requirements.

The conclusions referring to the above mentioned diagnosis were drawn mainly on the basis of students' remarks made in the course of working on the problems as well as on their answers given to open questions formulated in the interviews constructed for this study. Two of these interviews were conducted after completion of solving consecutive problems given in sets and directly concerned these problems (for example, I asked: *have you managed to solve this problem? Are you satisfied with this solution? do you think this problem is difficult? did you like the problems in this set?* etc.). The most significant conclusions referring to the students' motivations, however, were drawn from the answers to the questions of the third interview, which was conducted during the additional meeting with the students. It was based on the set of open questions aimed at creating the atmosphere of an unreserved discussion. Its objective was to induce the students (by asking questions like, for example, *do you like school? do you remember any event that took place during a maths lesson which was connected with solving a mathematical problem?*) to say something about themselves, their attitude toward studying, toward mathematics and, finally, toward solving mathematical problems. What deserves a short comment is the character of these questions. First of all, I would like to stress the fact that these questions were intentionally formulated in a very general way and did not refer to the subject of my study (it means that I never asked direct questions like, for example, do you like solving mathematical problems?). When asked directly about something a student does not necessarily give a fair answer. He or she often tries to say something what is more or less expected by the person asking the question. The existence of such a discrepancy (between the actual state and the comments of persons who are being examined) was confirmed by the studies of J. Mason.

The research I carried out confirmed the obvious fact that there is a wide range of possible reasons why a child is eager to learn. For example, a student studies because of a genuine interest in the subject or he or she studies a certain issue because it may be useful (like the ability to operate a computer). There may be a great number of motives why students decide to learn. They appear in various constellations, with various intensity and may influence the quality of both the process of learning and its results in a more or less positive way. What becomes a problem for a researcher attempting to diagnose students' motives for devoting their efforts to learn something is the fact that the degree, in which students themselves are aware of their own motives for these actions, varies considerably from individual to individual.

As a result of my investigations aimed at establishing the character of the emotional- motivational component of *students' attitudes toward mathematical problems* I obtained a typology of classes of student' motivations. It is, nonetheless, necessary to emphasize the following facts:

1. In the course of this study I did not succeed in separating the motiva-

tion for solving mathematical problems from the motivation for studying mathematics in general. For this reason, the conclusions I drew concern students' motivation for school study rather than just mathematics or solving problems.

2. The list of motivations I constructed does not exhaust all their types and it does not exhibit the features of a classification.
3. It is not possible to separate a unique type of motivation of a particular student; these types are interrelated, one of them may be contained in some other type etc. For example, a student studies in order to obtain good marks; but does it mean that this is the only reason? This student may also be interested in what he or she studies (and apart from that wants to obtain good marks).

In the course of the research presented here I distinguished the following types of motivations:

(1) *Social motivation*

   (a) *In the class group context* (a student studies because of the need to be accepted by his class group);
   (b) *In the peer group context* (a student wants to adjust himself or herself to a group of friends);
   (c) *In the teacher context* (a student studies in order to earn the teacher's positive attitude);
   (d) *In the family context* (a student studies to satisfy his or her parents);

(2) *Cognitive motivation*

   (a) *Intellectually-directed* (a student studies because he or she is interested in the subject);
   (b) *'Production'-directed* (a student enjoys solving similar problems and finds satisfaction in large quantity of them solved);

(3) *Moral motivation* (a student studies because he or she sees it as their duty)

(4) *Fear of the lack of progress*

   (a) *Punishment-directed* (a student studies for fear of failure at an exam, for fear of bad marks);
   (b) *Reward-directed* (a student studies in order to obtain good marks);

(5) *Prestige motivation* (a student studies to become better than others).

Within the frames of this presentation I do not intend to explain in detail the nature of the above- listed types of students' motivations to study, nor am I going to quote examples of students' comments and behavior, which became the source of my conclusions concerning a type of motivation a particular student had for studying mathematics. It is the consequence of the intention of this paper, which aims at putting special emphasis on the role which a proper motivation plays and should play in the process of assimilating knowledge by students. My research shows that although students display apparently similar cognitive behavior (which apparently suggests similar difficulties in solving problems) the reasons of emotional-motivational nature behind such a behavior may nonetheless be fundamentally different. Thereupon, if a teacher's estimation of students' work is based solely on observable activities it may happen that this teacher will never reveal the true reason of students' failures and therefore will not be able to undertake suitable re-educational actions.

It is also necessary to point out that modeling proper positive motivations for studying is one of the most crucial objectives of school. There are hierarchies of mathematics-teaching aims in which modeling and developing student's personality is indicated as the most vital objective, which means going far beyond the realm of mathematics ([4]). Therefore, recognizing the degree of realization of higher level aims of teaching mathematics becomes a relatively new and extremely complex goal of the theory as well as of practice of mathematics teaching. It is an extremely difficult task and particularly so with regard to mathematics, since the type of knowledge conveyed here seems to rule out the opportunity of conveying other values. The necessity of developing research activity in this direction was indicated by A. Bishop ([1]) and J. Moddleton and P. Spanias ([8]), who quoted statistical data that revealed a dramatic decline in interest in mathematics occurring in the course of teaching this subject in the USA. A. Bishop strongly emphasizes the influence of attitudes towards mathematics displayed by teachers on students. This proves the necessity of intensifying research activities in this direction.

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