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Stochastic and Statistic in Elementary School Teacher's Studies

Abstract. The article contents examples of mathematical education of stochastics and statistics in lessons for future elementary school teachers studying at the Pedagogical Faculty of University of Jan Evangelista Purkyne in Usti nad Labem. In the article are also examples of cooperation of the Pedagogical Fakulty in Usti nad Labem and the Pedagogical Academy in Krakow in terms of students education.

On the Czech seminar "Mathematics and Elementary School Teachers" which was held on 23 – 24 April 1998 on Pedagogical University in Hradec Králové I have informed about an introduction of statistic as new subject in studies of elementary school teachers at Pedagogical Faculty of J.E. Purkyně University in Ústí nad Labem.

Education of statistic, that is closely connected to stochastic (theory of probability calculus) is made in cooperation with the Institute of Mathematics of Pedagogical University in Cracow namely with the director of stochastic department Prof. Adam Plocki.

We focus on the coherence between stochastic and mathematical statistic. In stochastic we determine the course of event from its probable model whereas in statistic is the process contrary it means that features of the model are determined on basis of data spreading out from its realization.

And now example which is based on experiences from education in Poland and which contains our idea.

Exercise 1: Two players, each is throwing two coins. The task is to choose one side of the coin which the player guess will fall. For each right guess the player gets point. Who gets more points is the winner. Which choice is better to choose?

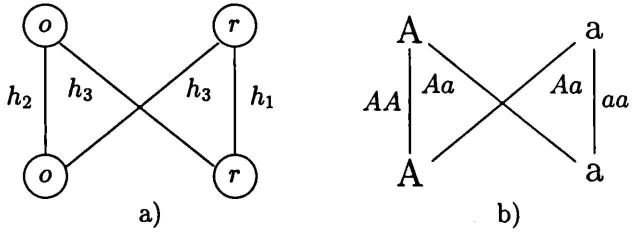
When throwing two coins three results are possible:
h_1: two reverses
h_2: two obverses
h_3: reverse and obverse

Set M is space of all results. $M = \{h_1, h_2, h_3\}$, each abscissa represents one result (see picture 1a).
Probability of result h_1 is $\frac{1}{4}$, probability of result h_2 is $\frac{1}{4}$ and probability of result h_3 is $\frac{1}{2}$.

Students play the game and check out that the best strategy is to typ result reverse and obverse.

Exercise 2: Mother has a genotype Aa. Father has the same genotype. What should be the genotype of their child? Child is overtaking one letter from mother's genotype and one from father's genotype. For each of the child's genotypes find probability of chance of rise. Genotypes Aa and aA are identical. (In genetics capital letter marks dominant gene and small letter marks recessive gene).

We again create model of connection of two various germinal cells (heterozygots) (see picture 1b).



Picture 1.

From picture 1 we can see that connection of two heterozygots can be simulated by throwing two coins.
Answer to our exercise is that probability of rise of child with genotype AA is $\frac{1}{4}$, with genotype aa is also $\frac{1}{4}$ and with genotype Aa is $\frac{1}{2}$.

Lets have a look at occurrence of genotypes in next generation and lets count its probability:

	AA 0,25	aa 0,25	Aa 0,5
AA 0,25	$AA \ldots 1$	$Aa \ldots 1$	$AA \ldots 0,5$ $Aa \ldots 0,5$
aa 0,25	$Aa \ldots 1$	$aa \ldots 1$	$aa \ldots 0,5$ $Aa \ldots 0,5$
Aa 0,5	$Aa \ldots 0,5$ $AA \ldots 0,5$	$aa \ldots 0,5$ $Aa \ldots 0,5$	$aa \ldots 0,25$ $Aa \ldots 0,50$ $AA \ldots 0,25$

Table 1.

We count probability of occurrence in new generation. We mark it f like frequency. It concerns independent cases.

$f(AA)$:

$f(AA) \cdot f(AA) \cdot 1 = 0,25 \cdot 0,25 \cdot 1 = 0,0625$
$f(AA) \cdot f(Aa) \cdot 0,5 = 0,25 \cdot 0,5 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(AA) \cdot 0,5 = 0,5 \cdot 0,25 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(Aa) \cdot 0,25 = 0,5 \cdot 0,5 \cdot 0,25 = 0,0625$

$f(AA) = 0,0625 + 0,0625 + 0,0625 + 0,0625 = 0,25$

$f(aa)$:

$f(aa) \cdot f(aa) \cdot 1 = 0,0625$
$f(aa) \cdot f(Aa) \cdot 0,5 = 0,25 \cdot 0,5 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(aa) \cdot 0,5 = 0,5 \cdot 0,25 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(Aa) \cdot 0,25 = 0,5 \cdot 0,5 \cdot 0,25 = 0,0625$

$f(aa) = 0,0625 + 0,0625 + 0,0625 + 0,0625 = 0,25$

$f(Aa)$:

$f(AA) \cdot f(aa) \cdot 1 = 0,25 \cdot 0,25 \cdot 1 = 0,0625$
$f(AA) \cdot f(Aa) \cdot 0,5 = 0,25 \cdot 0,5 \cdot 0,5 = 0,0625$
$f(aa) \cdot f(AA) \cdot 1 = 0,25 \cdot 0,25 \cdot 1 = 0,0625$
$f(aa) \cdot f(Aa) \cdot 0,5 = 0,25 \cdot 0,5 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(AA) \cdot 0,5 = 0,5 \cdot 0,25 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(aa) \cdot 0,5 = 0,5 \cdot 0,25 \cdot 0,5 = 0,0625$
$f(Aa) \cdot f(Aa) \cdot 0,5 = 0,5 \cdot 0,5 \cdot 0,5 = 0,1250$

$f(Aa) = 0,0625 + 0,0625 + 0,0625 + 0,0625 + 0,0625 + 0,0625 + 0,1250 = 0,5$

We make sure that probability of occurrence in new generation is in accord with occurrence in previous generation.

The result was proved by students in experiment. In one sack was one plastic coin (black on both sides) — genotype AA, one white coin — genotype aa and two coins (black on one side and white on other side) — genotype Aa (population of mothers). In second sack were the same coins (population of fathers). Students take out one coin from each sack and laid them on table. Colours they saw represented the gene of child. Results of the experiment were written down into a chart.

I also mentioned experiment of interbreeding bean plants which was made by Czech botanist J.G. Mendel (1822 - 1884). Gene A designates red colour of bean flower and gene a designates white colour. Plants with genotype Aa have pink flowers. During the first year of experiment all flowers were pink (see exercise 2 - Aa genotype of mother, aA genotype of father). Next year there were pink, red and white flowers. The number of red and white flowers was the same and pink flowers were two times more.

The lesson continued by determination of frequency of genotypes and fenotypes of blood-groups in population. This exercise helps students to understand the method of exclusion paternity in justice.
Chart of genotypes and fenotypes in population:

Genotype	Fenotype	Frequency of genotype	Frequency of fenotype
AA A0	A	0,08 0,33	0,41
BB B0	B	0,02 0,16	0,18
AB	AB	0,08	0,08
00	0	0,33	0,33

Table 2.

Before child's birth it is possible to designate its blood-group according to father's and mother's blood-groups.
Simply said: Child overtakes one letter from mother's genotype and one from father's genotype. Mathematically said: If mother's blood-group genotype is xx and father's blood-group genotype is xy than child should have blood-group xx in probability 0,5. Frequency of genotypes is $f_{xx} + f_{xy} = 1$.
See chart of genotypes and fenotypes of father, mother and child:

			mother					
			A		B		AB	00
			AA 0,08	A0 0,33	BB 0,02	B0 0,16	0,08	0,33
father	A	AA 0,08	AA...1	AB...0,5 A0...0,5	AB...1	AB...0,5 A0...0,5	AA...0,5 AB...0,5	A0...1
		A0 0,33	AA...0,5 A0...0,5	AA...0,25 A0...0,5 00...0,25	AB...0,5 B0...0,5	AB...0,25 A0...0,25 B0...0,25 00...0,25	AA...0,25 AB...0,25 A0...0,25 B0...0,25	A0...0,5 00...0,5
	B	BB 0,02	AB...1	AB...0,5 B0...0,5	BB...1	BB...0,5 B0...0,5	AB...0,5 BB...0,5	B0...1
		B0 0,16	AB...0,5 A0...0,5	AB...0,25 A0...0,25 B0...0,25 00...0,25	BB...0,5 B0...0,5	BB...0,25 B0...0,5 00...0,25	AB...0,25 A0...0,25 BB...0,25 B0...0,25	B0...0,5 00...0,5
	AB 0,08		AA...0,5 AB...0,5	AA...0,25 A0...0,25 B0...0,25 AB...0,25	AB...0,5 BB...0,5	AB...0,25 A0...0,25 BB...0,25 B0...0,25	AA...0,25 AB...0,5 BB...0,25	A0...0,5 B0...0,5
	00 0,33		A0...1	A0...0,5 00...0,5	B0...1	B0...0,5 00...0,5	A0...0,5 B0...0,5	00...1

Table 3.

In the chart we can see for example: If father's blood-group is AA and mother's blood-group is $A0$ child's blood-group should never be BB, $B0$, AB, and 00. Probability of occurrence of those blood-groups is zero. If father's blood-group is A and mother's blood-group is also A child will never have blood-group B or AB.

If we count probability of frequency of genotypes in new generation the result will be the same as in the previous generation.
e.g.: Frequency of genotype AA in population is 0,08. It means $f(AA) = 0,08$. Now we will count frequency of genotype AA in new generation.

$$f(AA) \cdot f(AA) \cdot 1 = 0,08 \cdot 0,08 \cdot 1 = 0,0064$$
$$f(AA) \cdot f(A0) \cdot 0,5 = 0,08 \cdot 0,33 \cdot 0,5 = 0,0132$$
$$f(AA) \cdot f(AB) \cdot 0,5 = 0,08 \cdot 0,08 \cdot 0,5 = 0,032$$
$$f(A0) \cdot f(AA) \cdot 0,5 = 0,33 \cdot 0,08 \cdot 0,5 = 0,0132$$
$$f(A0) \cdot f(A0) \cdot 0,25 = 0,33 \cdot 0,33 \cdot 0,25 = 0,027225$$
$$f(A0) \cdot f(AB) \cdot 0,25 = 0,33 \cdot 0,08 \cdot 0,25 = 0,0066$$
$$f(AB) \cdot f(AA) \cdot 0,5 = 0,08 \cdot 0,08 \cdot 0,5 = 0,0032$$
$$f(AB) \cdot f(A0) \cdot 0,25 = 0,08 \cdot 0,33 \cdot 0,25 = 0,0066$$
$$f(AB) \cdot f(AB) \cdot 0,25 = 0,08 \cdot 0,08 \cdot 0,25 = 0,0016$$

Add up frequencies of genotypes AA in new generation:

$$0,0064 + 0,0132 + 0,0032 + 0,0132 + 0,027225 + 0,0066 + 0,0032 + 0,0066 + 0,0016 = 0,081225 = 0,08$$

The frequency of genotype AA in new generation is 0,08 which is the same as in previous generation.

References

[1] J. Melichar, *Zavedení nového předmětu statistika ve studiu učitelství pro 1. stupeň ZŠ*, In: Matematika a učitelé 1. stupně základní školy, Gaudeamus, Hradec Králové 1998, page 55-58, ISBN 80-7041-830-3

[2] J. Melichar, *Realistic Approach Towards Mathematics Teaching*, In: Matematyka IV, Wyższa Szkoła Pedagogiczna w Częstochowie, 1998, 58-62.

[3] A. Płocki, *Pravděpodobnost kolem nás – počet pravděpodobnosti v úlohách a problémech*, Acta Universitatis Purkynianae 68, Ústí nad Labem 2001.

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