

# Professor Andrzej Zajtz a scholar and educator

on the 70th birthday anniversary

Differential geometry was formed almost simultaneously with mathematical analysis. Its brisk development came with the success of the General Relativity Theory, which made extensive use of tensor calculus. The first Pole to do research on differential geometry at the beginning of the 20th century was Kazimierz Żorawski from the Jagiellonian University. Antoni Hoborski, another Cracovian mathematician, took it as his main sphere of interest and dreamed of the “Polish School” of differential geometry. Unfortunately, the Second World War and his sudden death in a concentration camp put an end to realization of his dreams. Effective steps aiming at fulfilling Hoborski’s dream were undertaken by a student of his – Stanisław Gołąb. His interests revolved around the theory of geometric objects, which soon spread even beyond the Cracovian university centre. Professor Andrzej Zajtz proved himself a worthy successor of Professor’s Gołąb’s work in Kraków.

Andrzej Zajtz was born on December 9, 1934, in Radom. In 1955 he graduated from the Jagiellonian University, where he had studied mathematics. Even during his studies he worked as an assistant in the Institute of Mathematics of the AGH University of Science and Technology. In 1957 he took the position of an assistant in the Department of Geometry of the Jagiellonian University, whose head at time was Professor Gołąb. In 1961 he obtained a doctor’s degree in the fields of mathematics and physics and in 1966 a habilitation in mathematics. He worked as an associate professor in the Institute of Mathematics of the Jagiellonian University until 1980 when he was honoured a professor’s title. He was the head of the Geometry Department in the Institute of Mathematics of the Jagiellonian University for twenty years until 1990 when he moved to the Pedagogical University of Kraków. In 1994 Professor Zajtz became the head of the Department of Geometry and Differential Equations in the Institute of Mathematics of the Pedagogical University.

He frequently lectured at universities abroad. Between 1972 and 1975 he lectured at Ahmadu Bello University in Zaria, Nigeria, and from 1982 to 1983 at the Universidad Central de Venezuela in Caracas. He spent two years (1987-1989) as a professor at the Université de Tlemcen in Algeria and later (1990-1992) at the University of Zimbabwe in Harare. During his stay there, Professor Zajtz took an active part in the scientific and didactic work of these universities. Among his other duties, he acted as a consultant of various lecture programmes,

especially in the field of geometry. He inspired and consulted scientific work of younger colleagues. Among other results of his activity one may mention four PhD theses supervised by him. This cooperation is being continued till now.

His scientific activity concentrated at first on differential geometry under weak differentiability conditions, and then on the theory of geometric objects, which was the main stream of geometrical research in the Cracovian mathematical centre. Geometric objects integrate the definitions of such significant concepts as tensors and connections. Their classification and study of their features had a great impact not only on the differential geometry but also on theoretical and practical applications in physics. Professor Zajtz classified important families of geometric objects and examined their properties in specific cases (see for instance [2]-[5], [8], [9]).

The theory of geometric objects is closely related to the theory of functional equations. It is also in this field that Professor Zajtz made significant discoveries [1] – [4]. His interests embraced differential geometry at its highest contemporary level. At the end of the 1970's the theory of geometric objects gained a new global description in the form of natural bundles and the natural prolongation functors. The academic textbook "Foundations of differential geometry of natural bundles" (1984), in cooperation with M. Paluszny, compiled A. Zajtz's lectures in Caracas. It included the latest outcomes in the natural bundles theory as well as generalizations of several important theorems.

A. Zajtz determined the sharp estimation of the order of natural bundles [6]. This fundamental result completed the long history of the order problem for differential geometric objects (in a new formalism – of natural functors), considered, in particular, by Penzov and Gołab (1950's), up to the results of Palais-Terng and Epstein-Thurston (late 1970's). His estimation formula has found a direct application in the gauge-theory in theoretical physics. Further, he determined the sharp estimation of the order of natural functors on some basic local categories of manifolds (for instance manifolds with locally integrable volume form, symplectic manifolds and contact manifolds).

The natural differential operators theory has been developing together with the theory of natural functors. A. Zajtz investigated the problem of the row completion of natural operators and obtained some new general results [8]. Starting from a more general context, he contributed to the representation theory of certain groups of diffeomorphisms [7]-[9], [11]-[13]. He applied his own effective methods to the study of the equivalence and the order of certain types of representations. He also obtained positive results concerning the possibility of embedding diffeomorphisms in a smooth flow [14]. In parallel to these activities, but still within the general framework of his research, he proved some nonlinear Peetre-like theorems on local operators (such operators are very common in differential geometry) [10] – in his results, as opposed to previously formulated ones, references to the Whitney extension theorem are avoided.

The research outcomes of Professor Zajtz were presented in over 60 articles, both in Poland and abroad. He actively participated in many scientific conferences, where he delivered talks and was in charge of sessions or a member of scientific committees. In a short period of 1997-2001 his new original results were presented at ten conferences he attended.

Professor Zajtz was a supervisor of numerous MSc and thirteen PhD theses. Hitherto two of his former PhD students qualified as titular professors.

Scientific development of his numerous students participating in his seminars has always been Professor Zajtz's particular concern. He encourages them to study advanced problems and to search for and draw their own conclusions. He is willing to discuss new trends and latest discoveries in differential geometry. There was only one person at the Department of Geometry holding doctor's degree at the time when Professor Zajtz became the head of that Department but in just twenty-year's time (1990) this number increased by seven – all but one worked under his supervision.

During the 50 years of his work he has taught the majority of main mathematical subjects. Moreover, he lectured on mathematics at other university institutes. It is worth mentioning that his lectures on analysis were keenly appreciated by the students of physics. Between 1976-1979 he was the Deputy Director of the Institute of Mathematics of the Jagiellonian University. At that time he was also the head of the Cracovian Branch of the Polish Mathematical Society. Twice he became a member of the Senate Board for the cooperation with the secondary educational system. From 1983 to 1990 he tutored secondary school "university classes" (with an originally designed advanced course of mathematics). In spite of his diverse duties he has never rejected requests to take part in the Mathematical Work Sessions for the Primary and Secondary Schools as a juror and chairman.

Prof. Zajtz was awarded the three most important medals: The Knight's Cross of Poland's Restoration (Krzyż Kawalerski Orderu Odrodzenia Polski), The Gold Medal of Merit (Złoty Krzyż Zasługi) and the National Education Committee Medal (Medal Komisji Edukacji Narodowej) for his didactic, scientific and organizational work.

It is said that the proof of the man is in his actions. Professor Zajtz has proved to be an excellent mathematician with broad interests. He is known as a man of mark, demanding and just. What he appreciates among his students is independent thinking and resolution to pursue the scientific truth. He is able to encourage young people to investigate puzzling questions of mathematics and stimulate their interest in new spheres of geometry.

It is only fair to add that Professor Andrzej Zajtz's profound influence on the Cracovian geometry research centre enabled it to establish itself as one of the most significant among such centres in Poland.

Zdzisław Pogoda

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