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The First International Conference on Functional Equations in Poland: Zakopane 1967

Abstract. The meeting in 1967 was organized by Professors Stanisław Gołąb and Marek Kuczma. It was held in Tatra mountains and was the first international conference on functional equations organized in Poland. For most of Polish participants this was the first occasion to give a talk in a foreign language.

The article is based on the Conference Proceedings which were published in 1969, cf. [Z]. The participants, survey lectures and short communications are briefly reminded. Further international meetings on functional equations (and inequalities), up to the 6th ICFEI, are also mentioned.

The Sixth International Conference on Functional Equations and Inequalities (ICFEI) took place 30 years after the First International Conference on Functional Equations (ICFE) in Poland held at Zakopane, October 9-13, 1967 in the mountain hostel "Kalatówki". In fact, the next meeting like the ICFE was organized in Poland only in 1984 at Sielpia and it was the first in the series of ICFEI, organized by the Institute of Mathematics of the Pedagogical University in Kraków.

The aim of this article is to remind the meeting in Zakopane which had proved, among others, that already at that time Poland was (as it is at the present time) one of main centres of the development of the theory of functional equations and inequalities.

The ICFE was organized by the Chair of Functional Equations of the Katowice Branch of the Jagiellonian University (Professor Marek Kuczma) with the cooperation of the Chair of Geometry of the Jagiellonian University in Kraków (Professor Stanisław Gołąb). Miss Halina Światak acted as a secretary of the Conference. It was the third meeting on the subject which took place in Eastern Europe, after those in Hungary at Sárospatak (1963) and at Miskolc

(1966). It was also the eight such conference all over the world since, thanks to Professor János Aczél, from the year 1962 on there were organized annual (except for 1964) International Symposia on Functional Equations (ISFE), in 1962, 1963, 1965, 1966 at the Mathematisches Forschungsinstitut Oberwolfach (Germany) and the Fifth ISFE at Waterloo (Canada). The Scientific Committee of these ISFE consisted of Professors: János Aczél (Canada), Otto Haupt (Germany) and Aleksander Ostrowski (Switzerland). At that time travelling to *West* from Eastern Europe was an uneasy task. That is why another series of meetings was initiated by Hungarian colleagues.

The Proceedings of the ICFE were published in 1969 as a separate issue [Z] of the present *Universitatis Iagellonicae Acta Mathematica*. They contain the programme of the Conference and the foreword with the list of 47 participants, the texts of 8 expository lectures and of 14 papers, 7 abstracts of talks, the list of problems proposed by M. Hosszú and a report on solutions of one of his problems. There is also printed a picture taken of a group of participants which is reproduced here thanks to a kind permission of Professor Bolesław Szafirski, the present editor of the *Acta*. The names were attributed to the persons by the author with a help of Professor Zoltán Daróczy.



1st row: M. Kuczma, P. Fisher (H), S. Czerwik, M. Ochot.

2nd row: E. Vincze and Mrs. Vincze (H), Mrs. Daróczy (H), Z. Kareńska - ?, M. Rozmus-Chmura - ?, Mrs. Kuczma, I. Stamate (R), H. Świątak, D. V. Ionescu (R), I. Fenyő (H), S. Gołąb, Mrs. Fenyő (H), G. Majcher, B. Szociński.

3rd row: B. Choczewski, G. Knapcz (H), Z. Daróczy (H), D. Brydak, M. Bajraktar-
ević (Y), R. Djordjević (Y) - ?, E. Gesztelyi (H), S. Kurepa (Y), A. Climescu (R),
W. Maier (G), Z. Krzeszowiak, B. Barna (H), J. Tabor, S. Wołodzko.

G = *Germany*, H = *Hungary*, R = *Romania*, Y = *Yugoslavia*.

The other participants were: M. Hosszú (H), A. Moór (H); and from Poland: H. Adamczyk, L. Anczyk, Cz. Ginalski, A. Kapcia, W. Kleiner, J. Kordylewski, I. Kotlarski, M. Kucharzewski, J. S. Lipiński, J. Luchter, J. Matkowski, S. Midura, Z. Moszner, A. Smajdor, W. Smajdor, S. Topa, A. Zajtz. There were 16 foreign participants: 1 from East Germany (Jena), 9 from Hungary (Budapest 3, Debrecen 3, Miskolc 2, Szeged 1), 3 from Rumania (Cluj 2, Iași 1) and 3 from Yugoslavia (Beograd 1, Sarajevo 1, Zagreb 1). Several other invited guests from abroad were unable to attend the ICFE. Polish participants in the number of 31 came from Częstochowa (2), Gliwice (1), Katowice (7), Kraków (16), Łódź (1), Rzeszów (1), Świętochłowice (1) and Warszawa (2).

In the sequel we are going to remind some distinctive features of the programme of the ICFE and to report very briefly on the contents of the Proceedings [Z] of the meeting.

Professors S. Gołąb and M. Kuczma, the editors of the Proceedings wrote in the Foreword ([Z], 7-8) on the ICFE that its *programme (contrary to that of the previous meetings) contained, besides short communications reporting on recent own results, also a few longer lectures of the expository character* and that mainly because of the latter they found *it purposeful to publish in detail the proceedings of the conference*.

We first report on these expository papers in order of their appearance in the programme.

S. Gołąb, *Functional equations in geometry*, [Z], 13-19.

There is first given a short review of functional equations appearing in the foundations of geometry, non-Euclidean geometry, projective geometry, J. Aczél's axiomatic definitions of scalar and vector product of a couple of vectors in \mathbb{R}^3 and in the problem of determining the area of a triangle in a projective space. The Author excluded the functional equations of the theory of geometric objects, cf. [A-G]. Two problems on rotational body as well as one of defining the inner product in a vector space by means of the norm of the vector are also indicated. All three are not completely solved yet. [The latter is reformulated in the section of Problems and Remarks of the Report of Meeting of 6th ICFEI.]

S. Kurepa, *Functional equations in vector spaces*, [Z], 27-34.

This is a reach in results presentation of the theory of additive functions, quadratic functionals, exponential functions, cosine functional equation; treated in vector spaces. In particular, the Author quotes 11 of his papers, published in the decade 1955-65; the list of references consisting of 44 items.

M. Kuczma, *Problems of uniqueness in the theory of functional equations in a single variable*, [Z], 41-48.

Obviously, it is a fundamental problem of the theory. Instead of resuming the contents of the article, aimed at showing different approaches to the problem of determining a unique solution φ to a functional equation in a single variable (FESV), say,

$$\varphi = g \cdot \varphi \circ f + h$$

in a prescribed class of functions (based on the literature of 36 items), let us use M. Kuczma's own words concluding the paper ([Z], 46). *In my opinion it would be highly desirable to define, for a possibly large class of equations in a single variable, the principal solution, similarly as it has been done by E. Nörlund [for the difference equation of the first order] and by G. Szekeres (...) for Schröder's and Abel's equations. (...) I do not much hope to accomplish such a task in my life. But if reasearches of myself and of my colleagues and disciples help somebody to do this in the future, I shall feel that my life and my labour have not been lost in vain* (What has been done till 1990 in this direction may be found in the book by M. Kuczma, B. Choczewski and R. Ger [K-Ch-G]).

W. Maier, *Funktionalgleichungen mit analytischen Lösungen*, [Z], 57-62.

There are derived explicit formulae for complex analytic solutions of some functional equations connected with interpolation formula and with addition theorems.

B. Choczewski, *Topological methods in the theory of functional equations in a single variable*, [Z], 67-74.

Applications of Banach's and Schauder's fixed point theorems (FPT) in the theory are discussed. M. Bajraktarević (1957) was the first to obtain, via Banach's principle, a result on bounded solutions of a FESV of n -th order. Then, in the years 1961-67, for the equation

$$\varphi = H(\cdot, \varphi \circ f)$$

and its variants of higher orders, several existence and/or uniqueness results (in general, for smooth solutions) have been obtained with the use of the two FPT by M. Kuczma, J. Kordylewski, W. Smajdor, G. Majcher, H. Adamczyk and the author. A. Pelczar (1962, 1965) used a lattice-theoretical FPT to find extremal solutions of a FESV. An existence theorem for a FESV of infinite order has been proved by H. Adamczyk (1968) with the aid of the Tichonov FPT. The final claim of the article reads: *the possibilities of applications of topological methods in the domain of functional equations are limited, although*

they are still not exhausted (It soon appeared, in particular, that new FPT due to Boyd-Wong, Matkowski, Meir-Keeler helped much in the development of the theory, cf. [K-Ch-G]).

M. Kucharzewski, A. Zajtz, *Funktionalgleichungen mit Matrizenargumenten*, [Z], 87-97.

This article is devoted to the following functional equations:

$$F(B \cdot A) = F(B) \cdot F(A), \quad g(B \cdot A) = F(B)g(A) + g(B), \quad A, B \in \mathfrak{M}_{n \times n}$$

which determine linear geometric objects. Here $F : \mathfrak{M}_{n \times n} \rightarrow \mathfrak{M}_{m \times m}$, $g : \mathfrak{M}_{n \times n} \rightarrow \mathfrak{M}_{m \times 1}$ are unknown functions, and $\mathfrak{M}_{n \times k}$ stands for the family of all $n \times k$ matrices. The equations are also considered in a more general case — for mappings of a differential group L_n^s into another. There are described mostly the results from the years 1958-68 obtained by S. Gołab, Z. Kareńska, M. Kucharzewski, M. Kuczma, J. Luchter and A. Zajtz (23 items).

G. Majcher, *Applications des équations fonctionnelles dans la théorie des équations différentielles partielles*, [Z], 109-118.

Functional equations in a single variable help in solving the Goursat Problem (GP) for hyperbolic PDE. A method has been first developed, for the original GP, by A. Bielecki and J. Kiszyński (1956). A generalized GP involving the le Roux integral (following Authoress' dissertation (1965)) and GP for sample PDE of higher orders are also considered.

Z. Daróczy, *Funktionalgleichungen in der Informationstheorie*, [Z], 119-121.

The Author gives a concise survey of results on functional equations which characterize the Shannon entropy and the entropies of positive order. Entropies without probability are also mentioned (The role of functional equations in the theory of information is stressed and the results obtained (in the papers quoted) by J. Aczél and Z. Daróczy (1963), Z. Daróczy (1968), J. Kampé de Fériet and B. Forte (1967), are included as an essential part of the monograph [A-Da] by J. Aczél and Z. Daróczy).

What concerns short communications there is a visible distinction between the papers (11 items) presented by Polish participants and the talks (13 items) of our foreign guests.

Members of Professor Kuczma's school gave 8 talks on solutions of FESV. Consecutively, linear or nonlinear equations were discussed in classes of functions which are: differentiable (B. Choczewski), locally analytic (W. Smajdor), meromorphic (J. Matkowski), r -times continuously differentiable (L. An-

czyk), convex (M. Rozmus-Chmura), monotonic (A. Smajdor), sign preserving (S. Czerwik) (Let us note that in 1968 there appeared the first monograph [K] on FESV written by M. Kuczma in which he presented the state of the theory in 1967, and reported on the results of the above mentioned talks).

In **D. Brydak's** talk ([Z], 83-85) the notion of the *iterative stability* of (linear) FESV has been introduced for the first time. (One can say that thanks to his calling attention to Hyer's stability, also the stability theory of functional equations in several variables came into being, especially in the last decade, cf., in particular, the programmes of ISFE and ICFEI, and also [R-T]).

To the domain of FESV also belong the papers by M. Bajraktarević — on the equation $f \circ g \circ f = g$, and by I. Fenyő — on the equation of homogeneous functions: $f(x) = |x|^\lambda f\left(\frac{x}{|x|}\right)$, $x \in E^n$.

In the talks of our foreign guests (and of two Polish participants) there were discussed various problems for functional equations in several variables. Below we indicate the equation and/or the problem dealt with in the talk (Let us note that in 1967 there were already two monographs, published by J. Aczél in 1961 [A1] and in 1966 [A2] and dealing with functional equations of this kind. A new book by J. Aczél and J. Dhombres [A-Dh] present the state of the theory up to the end of eighties).

P. Fischer (Budapest): Continuous solutions of the equation he called *isomoment*

$$f\left(\frac{1}{n} \sum_{i=1}^n x_i^m\right) = \frac{1}{n} \sum_{i=1}^n |f(x_i)|^m \operatorname{sgn} f(x_i)$$

where x_i are nonnegative and m is a given positive real number.

H. Świątak (Kraków): Applications of Authoress' general theorems on the regularity of distributional solutions of some functional equations in two variables to solving a generalized Haruki equation

$$f(u+t, v+t) + f(u+t, v-t) + f(u-t, v+t) + f(u-t, v-t) = 4f(u, v).$$

A. Climescu (Iași): Theorems on extending solutions to polynomial functional equations (in the sense of universal algebra).

R. Djordjević (Beograd): General continuous solution (over \mathbb{C}) of the equation

$$f_1\left(x_1, \sum_{j=2}^n x_j\right) + \sum_{i=2}^{n-1} f_i\left(x_i, x_{i+1}, \sum_{j=i+2}^{n+i-1} x_j\right) = 0, \quad n \geq 4.$$

I. Stamate (Cluj): Integral-functional equations whose solutions are polynomials only.

S. Topa (Kraków): Determination of the homomorphisms of some representation of the differential group of order two in two-dimensional space.

D. V. Ionescu (Cluj): The only solutions of the differential-functional equation

$$[x_0, x_1, \dots, x_n; f] = \frac{1}{n!} f^{(n)} \left(\frac{x_0 + x_1 + \dots + x_n}{n+1} \right)$$

(with [...] denoting the n -th divided difference) are polynomials of the degree $n+1$.

In four talks functional equations did not appear explicitly. These were talks given by A. Moór (Szeged) and G. Knapecz (Budapest) — both on geometric objects and those given by E. Gesztelyi (Debrecen) and J. Kotlarski (Warszawa) — both on generalizations of the convolution.

The talk given by **M. Hosszú** (Debrecen) deserves a special attention. Its first part was about on some fairly general conditions for functions to be independent, whereas in the second part there are found four functional equations the Author proposes to solve and his partial results. The first of the equations (for $f : \mathbb{R} \rightarrow \mathbb{R}$) reads:

$$f(x+y-xy) + f(xy) = f(x) + f(y) \tag{H}$$

and it is now known as *Hosszú's equation*. In the Proceedings there are collected further results on equation (H) obtained during the ICFE by H. Świątak, Z. Daróczy and I. Fenyő.

When looking backward for the three decades that passed away from the ICFE we first note with great satisfaction that J. Aczél's ISFE were held each year, in 11 countries: Austria, Canada, Czech Republic, France, Germany, Hungary, Italy, Poland, Spain, Switzerland, and the U.S.A. The 35th ISFE (1997) was held at Graz (Austria) and the 36th — at Brno (The Czech Republic), in May 1998. We all in Poland, working in the domain of functional equations, were happy to be able to organize the 27th ISFE at Bielsko-Biała, Katowice and Kraków in that turning year 1989 and to have among us Professor Marek Kuczma delivering a talk... The 34th ISFE was again held in Poland, at Wisła-Jawornik and was organized by colleagues from the Institute of Mathematics of the Silesian University in Katowice, the local organizing committee being chaired by Professor Roman Ger.

Having told a little bit about history of the ISFE let us turn now to the meetings which followed the line of those held in Sárospatak, Miskolc and Zakopane and were organized in Hungary and Poland. In Hungary were held the Symposia on Functional Equations at Miskolc in 1968 and 1970 and the three International Symposia on Functional Equations and Inequalities: at Debrecen in 1973 and 1979 and at Noszvaj in 1986. In Poland we had the ICFEI at: Sielpia (1984), Szczawnica (1987), Koninki (1991), Krynica (1993) and Muszyna-Złockie (1995). Thus our 6th ICFEI should rather be numbered 9th, as it actually is a continuation of the three Hungarian meetings.

Three other international meetings on functional equations were organized in the period 1986-1994 within the frames of the official cooperation agreement between the Universities of Graz and Katowice. These were: *Seminar on Functional Equations Graz-Katowice*, Graz 1986, *Austrian-Polish Seminar on Functional Equations and Iteration Theory*, Graz 1991 (On a special session dedicated to the memory of Marek Kuczma lectures were delivered by Karol Baron, Bogdan Choczewski and Roman Ger) and *Polish-Austrian Seminar on Functional Equations and Iteration Theory*, Cieszyn 1994, cf. *Annales Mathematicae Silesianae* 8 (1994).

In view of these data our meeting is also 10th on functional equations organized in Poland (having counted the two ISFE and the *Seminar* at Cieszyn) and 51st ever organized in the world (7 meetings preceding the ICFE at Zakopane, 29 ISFE held in the years 1968-1996, 5 conferences organized in Hungary, 3 *Seminars* and 5 ICFEI) (Therefore the 50th meeting on functional equations was the 34th ISFE in 1996 at Wisła-Jawornik!).

Let me conclude the present report with a few remarks.

The period of 30 years means a generation. Only 9 participants of the ICFE at Zakopane were present on the 6th ICFEI (D. Brydak, B. Choczewski, S. Czerwik, Z. Daróczy, J. Matkowski, S. Midura, Z. Moszner, A. Smajdor, J. Tabor). The organizers of the Zakopane meeting passed away, Stanisław Gołąb in 1980 and Marek Kuczma in 1991. They left their scientific schools of functional equations in the state of progress which, fortunately, is continuing by their close students (and their students) not only in Kraków and Katowice but also in Bielsko-Biała, Gliwice, Rzeszów and Zielona Góra.

Most of Polish participants made on the ICFE their debut in front of an international audience, giving talks in foreign languages (I remember my stage fright when presenting the survey lecture...). It is a joyful situation that on the ICFEI there were always talks given by young colleagues for the first time at an international conference.

Direct cooperation links Polish centers of researches in Functional Equations, first of all, with those in Hungary and Austria. Thanks to the Inter-

national Symposia on Functional Equations, and also — in due proportion — to the International Conferences on Functional Equations and Inequalities worldwide personal contacts were established between specialists dealing with functional equations and inequalities and with related topics. These are important factors guaranteeing further development of these domains of researches.

References

- [A1] J. Aczél, *Vorlesungen über Funktionalgleichungen und ihre Anwendungen*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1961.
- [A2] J. Aczél, *Lectures on functional equations and their applications*, Mathematics in Science and Engineering **19**, Academic Press, New York and London, 1966.
- [A-Da] J. Aczél, Z. Daróczy, *On measures of information and their characterizations*, Academic Press, New York, London, San Francisco, 1975.
- [A-Dh] J. Aczél, J. Dhombres, *Functional equations in several variables: With applications to mathematics, information theory and to the natural and social sciences*, Encyclopedia of Mathematics and Its Applications **31**, Cambridge University Press, Cambridge – New York – Port Chester – Melbourne – Sydney, 1989.
- [A-G] J. Aczél, S. Gołąb, *Funktionalgleichungen der Theorie der geometrischen Objekte*, Monografie Mat. **39**, Polish Scientific Publishers, Warszawa, 1960.
- [K] M. Kuczma, *Functional equations in a single variable*, Monografie Mat. **46**, Polish Scientific Publishers, Warszawa, 1968.
- [K-Ch-G] M. Kuczma, B. Choczewski, R. Ger, *Iterative functional equations*, Encyclopedia of Mathematics and Its Applications **32**, Cambridge University Press, Cambridge, New York, Port Chester, Melbourne, Sydney, 1990.
- [R-T] Th. Rassias, J. Tabor (eds.), *Stability of mappings of Hyers-Ulam type*, Hadronic Press Inc., Palm Harbor, 1994.
- [Z] Zeszyty Naukowe Uniwersytetu Jagiellońskiego 223, Prace Matematyczne **14**. Międzynarodowa Konferencja Równań Funkcyjnych, International Conference on Functional Equations, Zakopane, 9.X.-13.X.1967, Uniwersytet Jagielloński, Kraków, 1970.

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